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变时滞非线性中立型微分方程的稳定性

黄明辉,赵国瑞
(广州城建职业学院,广东 广州 510925)

摘 要:利用 Banach 不动点定理,研究变时滞非线性中立型微分方程,并在一定的条件下构造适当的压缩映射,得到了方程零解渐近稳定的新条件. 之前,几乎所有的学者在利用 Banach 不动点定理研究变时滞非线性中立型微分方程时,都需要时滞 τ 二次可微且 $\tau'\neq 1$. 和大多数研究方法不相同,这些新条件不需要时滞 τ 二次可微,也不要求 $\tau'\neq 1$. 所得结论推广了已有文献中的相应结果,并给出了一个实例验证了所得结论的有效性.

关键词:非线性;变时滞;不动点定理;渐近稳定性

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Stability of nonlinear neutral differential equations with variable delays

HUANG Ming - hui, ZHAO Guo - rui

(Guangzhou City Construction College, Guangzhou 510925, P. R. C.)

Abstract:By using the fixed point theory, the nonlinear neutral differential equation with variable delays is studied, and appropriate contractive mappings are constructed under certain conditions. New conditions for asymptotic stability of zero solutions of the equation are obtained. Previously, almost all scholars used Banach fixed point theory to study nonlinear neutral differential equations with variable delays, requiring τ quadratic differentiability and $\tau'\neq 1$. Unlike most research methods, these conditions do not require a quadratic differentiability of delay τ and$\tau'\neq 1$. The results obtained generalize the corresponding results in the literature. A practical example is given to verify the validity of the conclusions.

Key words:nonlinear; variable delay; fixed point theorem; asymptotic stability

1 预备知识

时滞非线性微分方程的研究,长期以来一直受到广大研究者的关注[1-12]. Jin、Zhang 等学者利用不动点理论研究了微分方程的稳定性,并取得了一系列的研究成果[13-16]. 2011 年,文献[1]利用不动点理论,研究了时滞线性中立型微分方程

$$x'(t) = -\sum_{j=1}^{N} b_j(t)x(t-\tau_j(t)) + \sum_{j=1}^{N} c_j(t)x'(t-\tau_j(t)), \tag{1}$$

零解的渐近稳定性. 2012 年,文献[2]利用不动点理论,研究了时滞线性中立型积分微分方程

$$x'(t) = -\sum_{j=1}^{N} \int_{t-\tau_j(t)}^{t} a_j(t,s)x(s)ds + \sum_{j=1}^{N} c_j(t)x'(t-\tau_j(t)), \tag{2}$$

零解的渐近稳定性. 然而,上述结果的条件非常严格,要求 c 可微且 τ 二次可微, $\tau'(t)\neq 1$, $t\in[0,\infty)$. 受此

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作者简介:黄明辉(1988-),男,讲师,研究方向:微分方程与动力系统. E - mail: 249596697@ qq. com
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启发,本文考虑以下变时滞非线性中立微分方程零解的渐近稳定性

$$x'(t) = -\sum_{j=1}^{N}\int_{t-\tau_j(t)}^{t} a_j(t,s)x(s)ds + \frac{d}{dt}Q(t,x(t-\tau_1(t)),\cdots,x(t-\tau_N(t))) + G(t,x(t-\tau_1(t)),\cdots,x(t-\tau_N(t))), \tag{3}$$

及初始条件 $x(t) = \psi(t) \in C([m(t_0),t_0],R)$,对任意 $t_0 \geq 0$,有

$$m_j(t_0) = \inf\{t-\tau_j(t),t_0 \geq 0\},\ m(t_0) = \min\{m_j(t_0),1 \leq j \leq N\}.$$

2 主要结果

定义 1 对任意 $(t_0,\varphi) \in [0,\infty)\times C([m(t_0),t_0],R)$,若 $x \in C([m(t_0),\infty))$ 在 $[t_0,\infty)$ 上满足方程(3),且当 $t \in [m(t_0),t_0]$ 时, $x(t) = \varphi(t)$,则称 x 为方程(3)经过 (t_0,φ) 的解,记为 $x(t) = x(t,t_0,\varphi)$.

引理 1 方程(3)等价于

$$x'(t) = \sum_{j=1}^{N}B_j(t,t-\tau_j(t))(1-\tau_j'(t))x(t,t-\tau_j(t)) + \frac{d}{dt}\sum_{j=1}^{N}\int_{t-\tau_j(t)}^{t}B_j(t,s)x(s)ds + \frac{d}{dt}Q(t,x(t-\tau_1(t)),\cdots,x(t-\tau_N(t))) + G(t,x(t-\tau_1(t)),\cdots,x(t-\tau_N(t))). \tag{4}$$

其中 $B_j(t,s) = \int_t^s a_j(u,s)du, B_j(t,t-\tau_j(t)) = \int_t^{t-\tau_j(t)} a_j(u,t-\tau_j(t))du$.

对方程(3)给出下列假设:

(H1) $a_j \in C(R^+\times[m(t_0),\infty],R)$, $\tau_j \in C(R^+,R^+)$ 且可微,当 $t\to\infty$, $t-\tau_j(t)\to\infty$,其中 $j = 1,2,\cdots,N$.

(H2) $Q(t,x_1,\cdots,x_N),G(t,x_1,\cdots,x_N)$ 对 $x_1,\cdots,x_N$ 是全局 Lipschitz 连续函数,即存在正数 $L_1,\cdots,L_N$ 和 $K_1,\cdots,K_N$,

$$|Q(t,x_1,\cdots,x_N) - Q(t,y_1,\cdots,y_N)| \leq \sum_{j=1}^{N}K_j\|x_j-y_j\|,\ Q(t,0,\cdots,0) = 0,$$

$$|G(t,x_1,\cdots,x_N) - G(t,y_1,\cdots,y_N)| \leq \sum_{j=1}^{N}L_j\|x_j-y_j\|,\ G(t,0,\cdots,0) = 0,$$

(H3)存在连续函数 $h_j:[m(t_0),\infty]\to R$, $j = 1,2,\cdots,N$ 和常数 $\alpha \in (0,1)$,对 $t \geq 0$,

$$\sum_{j=1}^{N}K_j + \sum_{j=1}^{N}\int_{t-\tau_j(t)}^{t}|B_j(t,s)+h_j(s)|ds + \sum_{j=1}^{N}\int_0^t e^{-\int_s^t H(u)du}\{|B_j(s,s-\tau_j(s))+h_j(s-\tau_j(s))||1-\tau_j'(s)|+K_j|H(s)|+L_j\}ds + \sum_{j=1}^{N}\int_0^t e^{-\int_s^t H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u)+h_j(u)|du\right)ds \leq \alpha,$$

其中 $H(t) = \sum_{j=1}^{N}h_j(t)$.

定理 1 设(H1)-(H3)成立. 若 $\int_0^\infty H(s)ds\to\infty$,则方程(3)的零解渐近稳定.

证明 对任意 $t_0 \geq 0$,设 $K = \sup_{t\geq 0}\{e^{-\int_0^t H(s)ds}\}$. 对固定的 $\psi \in C([m(t_0),t_0],R)$,令

$$S_\psi = \{x \in C([m(t_0),\infty],R): t\to\infty, x(t)\to 0 \text{ 且 } x(t) = \psi(t), t \in [m(t_0),t_0]\},$$

且其范数为 $\|x\| = \max\{|x(t)|: m(t_0) \leq t \leq t_0\}$,则 S_ψ 是一个完备度量空间.

方程(4)两边同时乘以 $e^{\int_{t_0}^{t}H(s)ds}$,并从 t_0 到 t 积分,得

$$x(t)=\psi(t_0)e^{-\int_{t_0}^{t}H(u)du}+\sum_{j=1}^{N}\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}h_j(s)x(s)ds+$$
$$\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}\sum_{j=1}^{N}B_j(s,s-\tau_j(s))(1-\tau_j'(s))x(s-\tau_j(s))ds+$$
$$\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}\sum_{j=1}^{N}\frac{d}{ds}\int_{s-\tau_j(s)}^{s}B_j(s,u)x(u)duds+$$
$$\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}\frac{d}{ds}Q(s,x(s-\tau_1(s)),\cdots,x(s-\tau_N(s)))ds+$$
$$\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}G(s,x(s-\tau_1(s)),\cdots,x(s-\tau_N(s)))ds,$$

通过分部积分并整理,得

$$x(t)=\{\psi(t_0)-\sum_{j=1}^{N}\int_{t_0-\tau_j(t_0)}^{t_0}B_j(t_0,s)\psi(s)ds-\sum_{j=1}^{N}\int_{t_0-\tau_j(t_0)}^{t_0}h_j(s)\psi(s)ds-$$
$$Q(t_0,\psi(t_0-\tau_1(t_0)),\cdots,\psi(t_0-\tau_N(t_0)))\}e^{-\int_{s}^{t}H(u)du}+$$
$$Q(t,x(t-\tau_1(t)),\cdots,x(t-\tau_N(t)))+\sum_{j=1}^{N}\int_{t-\tau_j(t)}^{t}[B_j(t,s)+h_j(s)]x(s)ds+$$
$$\sum_{j=1}^{N}\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}(B_j(s,s-\tau_j(s))+h_j(s-\tau_j(s)))(1-\tau_j'(s))x(s-\tau_j(s))ds+$$
$$\sum_{j=1}^{N}\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}(G(s,x(s-\tau_1(s)),\cdots,x(s-\tau_N(s)))-H(s)\times$$
$$Q(s,x(s-\tau_1(s)),\cdots,x(s-\tau_N(s))))ds-$$
$$\sum_{j=1}^{N}\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}H(s)\left(\int_{s-\tau_j(s)}^{s}[B_j(s,u)+h_j(u)]x(u)du\right)ds:=I_6,\tag{5}$$

定义映射 $P:S_\psi\to S_\psi$:对任意 $t\in[m(t_0),t_0]$, $(P\varphi)(t)=\psi(t)$,当 $t\geq t_0$,

$$(P\varphi)(t)=\{\psi(t_0)-\sum_{j=1}^{N}\int_{t_0-\tau_j(t_0)}^{t_0}B_j(t_0,s)\psi(s)ds-\sum_{j=1}^{N}\int_{t_0-\tau_j(t_0)}^{t_0}h_j(s)\psi(s)ds-$$
$$Q(t_0,\psi(t_0-\tau_1(t_0)),\cdots,\psi(t_0-\tau_N(t_0)))\}e^{-\int_{s}^{t}H(u)du}+$$
$$Q(t,\varphi(t-\tau_1(t)),\cdots,\varphi(t-\tau_N(t)))+\sum_{j=1}^{N}\int_{t-\tau_j(t)}^{t}[B_j(t,s)+h_j(s)]\varphi(s)ds+$$
$$\sum_{j=1}^{N}\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}(B_j(s,s-\tau_j(s))+h_j(s-\tau_j(s)))(1-\tau_j'(s))\varphi(s-\tau_j(s))ds+$$
$$\sum_{j=1}^{N}\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}(G(s,\varphi(s-\tau_1(s)),\cdots,\varphi(s-\tau_N(s)))-H(s)\times$$
$$Q(s,\varphi(s-\tau_1(s)),\cdots,\varphi(s-\tau_N(s))))ds-$$
$$\sum_{j=1}^{N}\int_{t_0}^{t}e^{-\int_{s}^{t}H(u)du}H(s)\left(\int_{s-\tau_j(s)}^{s}[B_j(s,u)+h_j(u)]\varphi(u)du\right)ds:=\sum_{j=1}^{6}I_j,\tag{6}$$

显然, $(P\varphi)\in C([m(t_0),\infty],R)$. 现在证明当 $t\to\infty$时, $(P\varphi)(t)\to 0$. 由于 $t\to\infty$时, $\varphi(t)\to 0$ 和 $t-\tau_j(t)\to\infty$. 因此,对任意 $\varepsilon>0$,存在 $T_1>t_0$,使得当 $s\geq T_1$,有 $|\varphi(s-\tau_j(s))|<\varepsilon,j=1,2,\cdots,N$. 因此,当 $t\geq T_1$

,(6)中的最后一项 I_6 满足

$$|I_6|\le\sum_{j=1}^{N}\int_{t_0}^{T_1}e^{-\int_s^t H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u)+h_j(u)||\varphi(u)|du\right)ds+$$

$$\sum_{j=1}^{N}\int_{T_1}^{t}e^{-\int_s^t H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u)+h_j(u)||\varphi(u)|du\right)ds\le$$

$$\sup_{\sigma\ge m(t_0)}|\varphi(\sigma)|\sum_{j=1}^{N}\int_{t_0}^{T_1}e^{-\int_s^t H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u)+h_j(u)|du\right)ds+$$

$$\varepsilon\sum_{j=1}^{N}\int_{T_1}^{t}e^{-\int_s^t H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u)+h_j(u)|du\right)ds.$$

此外,存在 $T_2\ge T_1$,使得当 $t\ge T_2$,

$$\sup_{s\ge m(t_0)}|\varphi(\sigma)|\sum_{j=1}^{N}\int_{t_0}^{T_1}e^{-\int_s^t H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u)+h_j(u)|du\right)ds=$$

$$\sup_{s\ge m(t_0)}|\varphi(\sigma)|e^{-\int_{T_1}^t H(u)du}\sum_{j=1}^{N}\int_{t_0}^{T_1}e^{-\int_s^{T_1} H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u)+h_j(u)|du\right)ds<\varepsilon,$$

由(H3)知,$|I_6|<\varepsilon+\alpha\varepsilon<2\varepsilon$. 因此,当 $t\to\infty$时, $I_6\to 0$. 同样地,可以证明当 $t\to\infty$时,(6)中其它项也趋向于零. 因此,当 $t\to\infty$时, $(P\varphi)(t)\to 0$,故 $P\varphi\in S_\psi$.

设任意 $\varphi,\psi\in S_\psi$,当 $t\ge t_0$ 时,

$$|(P\varphi)(t)-(P\psi)(t)|\le\left(\sum_{j=1}^{N}K_j+\sum_{j=1}^{N}\int_{t-\tau_j(t)}^{t}|B_j(t,s)+h_j(s)|ds+\right.$$

$$\sum_{j=1}^{N}\int_{0}^{t}e^{-\int_s^t H(u)du}\{|B_j(s,s-\tau_j(s))+h_j(s-\tau_j(s))||1-\tau_j{}'(s)|+K_j|H(s)|+L_j\}ds+$$

$$\left.\sum_{j=1}^{N}\int_{0}^{t}e^{-\int_s^t H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u)+h_j(u)|du\right)ds\right)\|\varphi-\psi\|,$$

由条件(H3)可得,P 是一个压缩系数为 α 的压缩映射. 所以,由压缩映射原理得,P 在空间 S_ψ 上存在唯一不动点 $x(t)$,它是方程(3)的解. 且 $x(t)$ 满足当 $t\in[m(t_0),t_0]$, $x(t)=\psi(t)$,当 $t\to\infty$时, $x(t,t_0,\psi)\to 0$.

为了证明渐近稳定性,需要证明方程(3)的零解是稳定的. 假设给定任意 $\varepsilon>0$ 和 $\delta>0(\varepsilon<\delta)$ 满足 $2\delta Ke^{-\int_0^{t_0}H(u)du}+\alpha\varepsilon<\varepsilon$. 如果 $x(t)=x(t,t_0,\psi)$ 是方程(3)的一个解,其中 $\|\psi\|<\delta$,对任意 $t\ge t_0$, $x(t)=(Px)(t)$. 下面证明 $t\ge t_0$, $|x(t)|<\varepsilon$.

显然,当 $s\in[m(t_0),t_0]$,有 $|x(s)|<\varepsilon$. 如果存在 $t^*>t_0$,使得 $x(t^*)=\varepsilon$,且当 $m(t_0)\le s<t^*$,有 $|x(s)|<\varepsilon$,则由(6)得

$$x(t^*)\le\|\psi\|\left(1+\sum_{j=1}^{N}\int_{t_0-\tau_j(t_0)}^{t_0}|B_j(t_0,s)+h_j(s)|ds+\sum_{j=1}^{N}K_j\right)e^{-\int_{t_0}^{t^*}H(u)du}+$$

$$\varepsilon\sum_{j=1}^{N}K_j+\varepsilon\sum_{j=1}^{N}\int_{t^*-\tau_j(t^*)}^{t^*}|B_j(t,s)+h_j(s)|ds+$$

$$\varepsilon\sum_{j=1}^{N}\int_{t_0}^{t^*}e^{-\int_s^{t^*}H(u)du}(|B_j(s,s-\tau_j(s))+h_j(s-\tau_j(s))||(1-\tau_j{}'(s))|+K_j|H(s)|+L_j)ds+$$

$$\varepsilon\sum_{j=1}^{N}\int_{t_0}^{t}e^{-\int_s^{t^*}H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u)+h_j(u)|du\right)ds\le$$

$$2\delta K e^{-\int_0^{t_0} H(u)du} + \alpha\varepsilon < \varepsilon.$$

这与 t^* 的定义相矛盾. 这说明,如果 $\int_0^\infty H(s)ds \to \infty$成立,方程(3)的零解渐近稳定.

3　应用举例

例 1　考虑以下变时滞非线性中立微分方程

$$x'(t) = -\sum_{j=1}^{2}\int_{t-\tau_j(t)}^{t} a_j(t,s)x(s)ds + \frac{d}{dt}Q(t,x(t-\tau_1(t)),x(t-\tau_2(t))) + G(t,x(t-\tau_1(t)),x(t-\tau_2(t))). \tag{7}$$

其中 $\tau_1(t) = 0.489t, \tau_2(t) = 0.478t, a_1(t,s) = 0.48/(s^2+1), a_2(t,s) = 0.52/(s^2+1)$,
$Q(t,x,y) = 0.072\sin(x/2) + 0.036\sin(y/3), G(t,x,y) = 0$.

证明　选取 $h_1(t) = 0.52t/(t^2+1), h_2(t) = 0.48t/(t^2+1)$,则 $H(t) = t/(t^2+1)$,

通过简单的计算可得, $K_1 = 0.036, K_2 = 0.012, L_1 = L_2 = 0, \sum_{j=1}^{2} K_j = 0.048$,

$$B_1(t,s) = \int_t^s \frac{0.48}{s^2+1}du = \frac{0.48(s-t)}{s^2+1}, B_2(t,s) = \int_t^s \frac{0.52}{s^2+1}du = \frac{0.52(s-t)}{s^2+1},$$

$$\sum_{j=1}^{N}\int_{t-\tau_j(t)}^{t} |B_j(t,s) + h_j(s)|ds = \int_{0.511t}^{t}\frac{s-0.48t}{s^2+1}ds + \int_{0.522t}^{t}\frac{s-0.52t}{s^2+1}ds =$$

$$1 - 0.48/0.511 - 0.52/0.522 - \ln(0.511\times 0.522) < 0.386,$$

$$\sum_{j=1}^{N}\int_0^t e^{-\int_s^t H(u)du}|H(s)|\left(\int_{s-\tau_j(s)}^{s}|B_j(s,u) + h_j(u)|du\right)ds < 0.386,$$

$$\sum_{j=1}^{N}\int_0^t e^{-\int_s^t H(u)du}\{|B_j(s,s-\tau_j(s)) + h_j(s-\tau_j(s))||1-\tau_j{}'(s)| + K_j|H(s)| + L_j\}ds <$$

$$\left(1-\frac{0.48}{0.511}\right)\int_0^t e^{-\int_s^t \frac{u}{u^2+1}du}\frac{s}{s^2+1/0.511^2}ds + 0.036\int_0^t e^{-\int_s^t \frac{u}{u^2+1}du}\frac{s}{s^2+1}ds +$$

$$\left(1-\frac{0.52}{0.522}\right)\int_0^t e^{-\int_s^t \frac{u}{u^2+1}du}\frac{s}{s^2+1/0.522^2}ds + 0.012\int_0^t e^{-\int_s^t \frac{u}{u^2+1}du}\frac{s}{s^2+1}ds <$$

$$1 - \frac{0.48}{0.511} + 0.036 + 1 - \frac{0.52}{0.522} + 0.012 < 0.1125.$$

因此, $\alpha = 0.048 + 0.386 + 0.386 + 0.1125 = 0.9325 < 1$,定理 1 中的所有条件成立. 由定理 1 可得方程(7)零解是渐近稳定的.

4　结论

本文利用不动点理论,研究变时滞非线性中立型微分方程零解的渐近稳定性. 所研究的方程引入了 $\frac{d}{dt}Q(t,x(t-\tau_1(t)),\cdots,x(t-\tau_N(t)))$ 和 $G(t,x(t-\tau_1(t)),\cdots,x(t-\tau_N(t)))$,比文献[1-2]中的方程更加一般化. 并且文献[1-2]的定理要求时滞 τ 二次可微, $\tau'(t)\neq 1$, $t\in[0,\infty)$,但本文定理 1 中仅要求 τ 连续可微,进一步削弱了对时滞 τ 的要求,从而推广了文献[1-2]的相应结果.

参考文献

[1] ARDJOUNI A. Fixed points and stability in linear neutral differential equations with variable delays[J]. Nonlinear Analysis, 2011(74): 2062 - 2070.

[2] ARDJOUNI A, DJOUDI A, SOUALHIA I. Stability for linear neutral integro - differential equations with variable delays[J]. Electronic Journal of Differential Equations, 2012, 2012(172): 1 - 14.

[3] ARDJOUNI A. Existence, uniqueness and positivity of solutions for a neutral nonlinear periodic differential equation[J]. Computational & Applied Mathematics, 2015, 34(1): 17 - 27.

[4] ARDJOUNI A. Stability in totally nonlinear neutral differential equations with variable delay using fixed point[J]. Proyecciones Journal of Mathematics, 2015, 34: 25 - 44.

[5] 黄明辉. 带可积时滞的非线性中立型微分方程周期解的存在性[J]. 广东工业大学学报, 2014, 31(4): 65 - 68.

[6] 黄明辉. 多时滞的非线性微分方程的渐近稳定性[J]. 广东工业大学学报, 2016, 33(1): 62 - 66.

[7] MESMOULI M B. Periodic solutions and stability in a nonlinear neutral system of differential equations with infinite delay[J]. Boletín de la Sociedad Matemática Mexicana, 2018, 24(1): 239 - 255.

[8] MESMOULI M B. Periodicity and non - negativity of solutions for nonlinear neutral differential equations with variable delay via fixed point theorems[J]. Mathematical Methods in the Applied Sciences, 2016, 39(11): 2840 - 2852.

[9] 钟越. 一类非线性 Volterra 方程零解的稳定性[J]. 数学物理学报, 2015, 35(5): 878 - 883.

[10] BURTON T A. Stability by fixed point methods for highly nonlinear delay equations[J]. International Journal on Fixed Point Theory Computation & Applications, 2004, 1(1): 3 - 20.

[11] 董彪, 何永春. 一类三阶中立型时滞微分方程的渐近稳定性[J]. 数学的实践与认识, 2014, 44(2): 199 - 204.

[12] 姚洪兴. 一类四阶时滞微分方程的渐进稳定性[J]. 江苏大学学报(自然科学版), 2010, 31(5): 616 - 620.

[13] ZHANG B. Fixed points and stability in differential equations with variable delays[J]. Nonlinear Anal, 2005, 63(5): 233 - 242.

[14] JIN C H, LUO J W. Asymptotic stability of differential equations with several delays[J]. Publ Math Debrecen, 2011, 78(1): 89 - 102.

[15] JIN C H, LUO J W. Stability of anintegro - differential equation[J]. Computers and Mathematics with Applications, 2009, 57(7): 1080 - 1088.

[16] JIN C H, LUO J W. Stability in functional differential equations established using fixed point theory[J]. Nonlinear Analysis, 2008, 68(11): 3307 - 3315.

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