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Schrödinger 方程的周期孤立波解及其时空分叉

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摘　要:扩展了 Hirota 法,即将 Hirota 法中的测试函数改用含有三角函数,双曲函数和指数函数的三波函数来替代,把一个非线性微分方程的求解转化为一个多项式方程组的求解,再利用 MATLAB 便能解决.把该方法用到 Schrödinger 方程,得到了其新的周期孤波解和周期双孤立波解等重要结果,进而研究 Schrödinger 方程的周期孤波解和周期双孤立波解所描述的动力系统的时空分岔问题.

关键词:Schrödinger 方程; Hirota 方法;双周期波解;孤立波解

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Periodic solitary wave solutions and its spatio – temporal bifurcation for Schrödinger equation

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Abstract: The Hirota method is extended by replacing the test function in Hirota method with the three – wave function containing trigonometric function, hyperbolic function and exponential function, and converting a linear differential equation solution into a polynomial solution, and then Matlab is used to solve it. By applying this method to Schrödinger equation, some new periodic solitary wave solutions and periodic double solitary wave solutions are obtained, and the time – space bifurcations of dynamical systems described by periodic solitary wave solutions and periodic double solitary wave solutions for Schrödinger equation are studied.

Key words: Schrödinger equation; Hirota method; double periodic wave solution; soliton wave solution

Schrödinger 方程是 1926 年建立的量子力学基本方程,在量子力学中有着极其重要的位置.求 Schrödinger 方程的精确解是很难而且是一个很重要的研究,从而吸引了一大批学者不断寻找求解的方法,现有的常见方法有 Inverse scattering 法[1], Backlund 法[2], Darboux transformation 法[3], Hirota 双线性法[4-6], F-展开法[7-8],齐次平衡法[9], Jacobi 椭圆函数展开法[10],包络变换法[11-12],ADM 方法[13]和指数函数法[14]及双指数函数法等.这些方法还不足以求解所有波方程,所以不少科学家们还在不断深入研究当中.

本文扩展了 Hirota 法,并用该法得到 Schrödinger 方程

$$-iu_t+\alpha u_{xx}+\beta|u|^2u=0, \tag{1}$$

的新的周期孤波解和周期双孤波解,从而讨论了它的动力系统的时空分岔问题.下面,给出这个方法及步骤.

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1 Schrödinger 方程的双线性变换

设 Schrödinger 方程有如下形式的解:

$$u(x,t) = \exp(i(\mathrm{P}x - \Omega t))\varphi(x,t). \tag{2}$$

把式(2)代入方程(1)得

$$(2\alpha \mathrm{P}\varphi_x - \varphi_t)i + (\Omega\varphi + \alpha\varphi_{xx} - \alpha \mathrm{P}^2\varphi + \beta\varphi^3) = 0, \tag{3}$$

把方程(3)的实部、虚部分开得

$$2\alpha \mathrm{P}\varphi_x - \varphi_t = 0, \tag{4}$$

$$\Omega\varphi + \alpha\varphi_{xx} - \alpha \mathrm{P}^2\varphi + \beta\varphi^3 = 0. \tag{5}$$

假设

$$\varphi(x,t) = \pm\sqrt{-\frac{2\alpha}{\beta}}(\ln f(x,t))_x, \tag{6}$$

把式(6)代入方程(4)-(5),整理可得如下两个双线性型:

$$2\alpha \mathrm{P}(f_{xx}f - f_x^2) - f_{xt}f + f_xf_t = 0, \tag{7}$$

$$(\Omega - \alpha \mathrm{P}^2)f_xf + \alpha f_{xxx}f - 3\alpha f_{xx}f_x = 0. \tag{8}$$

假设双线性型(7)-(8)的 $f(x,t)$ 能表示为如下形式:

$$f(x,t) = a_1\cos\xi_1 + a_2\sinh\xi_2 + \exp(-\xi_3) + a_3\exp\xi_3. \tag{9}$$

其中, $\xi_i = k_ix + w_it\,(i = 1,2,3)$, $k_i, w_i\,(i = 1,2,3)$ 是待定常数.

2 Schrödinger 方程的时空分岔

把式(9)代入方程(7)-(8),整理得到一个关于 $\sin(\xi_1)\exp(-\xi_3)$, $\cos(\xi_1)\exp(-\xi_3)$, $\sinh(\xi_2)\exp(-\xi_3)$, $\cosh(\xi_2)\exp(-\xi_3)$, $\sin\xi_1\sinh\xi_2$, $\cos\xi_1\cosh\xi_2$, $\sin(\xi_1)\exp(\xi_3)$, $\cos(\xi_1)\exp(\xi_3)$, $\sinh(\xi_2)\exp(\xi_3)$, $\cosh(\xi_2)\exp(\xi_3)$ 的两个方程,并令其系数和常数项均为零,得到一个关于 $\mathrm{P}, \Omega, a_i, k_i, w_i\,(i = 1,2,3)$ 超越代数方程组如下:

$$a_1(4\alpha \mathrm{P}k_1k_3 - k_1w_3 - k_3w_1) = 0, \tag{10}$$

$$a_1(-k_1w_1 + k_3w_3 + 2\alpha \mathrm{P}k_1^2 - 2\alpha \mathrm{P}k_3^2) = 0, \tag{11}$$

$$a_2(k_2w_3 + k_3w_2 - 4\alpha \mathrm{P}k_2k_3) = 0, \tag{12}$$

$$a_2(k_2w_2 + k_3w_3 - 2\alpha \mathrm{P}k_2^2 - 2\alpha \mathrm{P}k_3^2) = 0, \tag{13}$$

$$4a_3(-2\alpha \mathrm{P}k_3^2 + k_3w_3) + a_1^2(2\alpha \mathrm{P}k_1^2 - k_1w_1) + a_2^2(2\alpha \mathrm{P}k_2^2 - k_2w_2) = 0, \tag{14}$$

$$a_1a_2(k_2w_2 - k_1w_1 + 2\alpha \mathrm{P}k_1^2 - 2\alpha \mathrm{P}k_2^2) = 0, \tag{15}$$

$$a_1a_2(k_1w_2 + k_2w_1 - 4\alpha \mathrm{P}k_1k_2) = 0, \tag{16}$$

$$a_1a_3(k_3w_3 - k_1w_1 + 2\alpha \mathrm{P}k_1^2 - 2\alpha \mathrm{P}k_3^2) = 0, \tag{17}$$

$$a_1a_3(k_3w_1 + k_1w_3 - 4\alpha \mathrm{P}k_1k_3) = 0, \tag{18}$$

$$a_2a_3(k_3w_3 + k_2w_2 - 2\alpha \mathrm{P}k_2^2 - 2\alpha \mathrm{P}k_3^2) = 0, \tag{19}$$

$$a_2a_3(4\alpha \mathrm{P}k_2k_3 - k_2w_3 - k_3w_2) = 0, \tag{20}$$

$$(2\alpha k_3^2 + \alpha \mathrm{P}^2 - \Omega)k_3 = 0, \tag{21}$$

$$a_1k_1(\Omega-\alpha P^2-\alpha k_1^2-3\alpha k_3^2)=0,\tag{22}$$

$$a_1k_3(\Omega-\alpha P^2+\alpha k_3^2+3\alpha k_1^2)=0,\tag{23}$$

$$a_2k_2(\Omega-\alpha P^2+\alpha k_2^2-3\alpha k_3^2)=0,\tag{24}$$

$$a_2k_3(\alpha P^2-\Omega-\alpha k_3^2+3\alpha k_2^2)=0,\tag{25}$$

$$a_1^2k_1(\alpha P^2-\Omega-2\alpha k_1^2)=0,\tag{26}$$

$$a_1a_2k_1(\alpha P^2-\Omega+\alpha k_1^2+3\alpha k_2^2)=0,\tag{27}$$

$$a_1a_2k_2(\Omega-\alpha P^2+\alpha k_2^2+3\alpha k_1^2)=0,\tag{28}$$

$$a_2^2k_2(\Omega-\alpha P^2-2\alpha k_2^2)=0,\tag{29}$$

$$a_1a_3k_1(\alpha P^2-\Omega+\alpha k_1^2+3\alpha k_3^2)=0,\tag{30}$$

$$a_1a_3k_3(\Omega-\alpha P^2+\alpha k_3^2+3\alpha k_1^2)=0,\tag{31}$$

$$a_2a_3k_2(\Omega-\alpha P^2+\alpha k_2^2-3\alpha k_3^2)=0,\tag{32}$$

$$a_2a_3k_3(\Omega-\alpha P^2+\alpha k_3^2-3\alpha k_2^2)=0,\tag{33}$$

$$a_3^2k_3(\Omega-\alpha P^2-2\alpha k_3^2)=0.\tag{34}$$

利用数学软件 MATLAB 求解以上超越代数方程组,可以得到很多组解,然后选取了 5 组解进行讨论,该 5 组解能得到 Schrödinger 方程不同类型的解,其中,包括双孤立波解、周期双孤立波解和周期三孤立波解.

情形 I:

$$\begin{aligned}&a_1=0,\ a_2=a_2,\ a_3=a_3,\ P=P,\ k_1=k_1,\ k_2=\pm k_3,\ k_3=k_3,\\&\Omega=\alpha P^2+2\alpha k_3^2,\ w_1=w_1,\ w_2=\pm 2\alpha Pk_3,\ w_3=2\alpha Pk_3.\end{aligned}\tag{35}$$

把式(35)代入式(9)得

$$f(x,t)=a_2\sinh(\pm k_3x\pm 2\alpha Pk_3t)+\exp(-k_3x-2\alpha Pk_3t)+a_3\exp(k_3x+2\alpha Pk_3t).\tag{36}$$

①当 $a_3>0$ 时,令 $\xi_0=\ln\sqrt{a_3}$, 则式(36)可以改写为

$$f(x,t)=a_2\sinh(\pm k_3x\pm 2\alpha Pk_3t)+2\sqrt{a_3}\cosh(k_3x+2\alpha Pk_3t+\xi_0).\tag{37}$$

把式(37)和式(6)代入式(2)可得方程(1)的双孤波解为

$$u_1(x,t)=\pm\sqrt{-\frac{2\alpha}{\beta}}\frac{a_2k_3\cosh(k_3x+2\alpha Pk_3t)+2\sqrt{a_3}k_3\sinh(k_3x+2\alpha Pk_3t+\xi_0)}{a_2\sinh(k_3x+2\alpha Pk_3t)+2\sqrt{a_3}\cosh(k_3x+2\alpha Pk_3t+\xi_0)},\\ \exp(i(Px-(\alpha P^2+2\alpha k_3^2)t)).\tag{38}$$

$$u_2(x,t)=\pm\sqrt{-\frac{2\alpha}{\beta}}\frac{-a_2k_3\cosh(-k_3x-2\alpha Pk_3t)+2\sqrt{a_3}k_3\sinh(k_3x+2\alpha Pk_3t+\xi_0)}{a_2\sinh(-k_3x-2\alpha Pk_3t)+2\sqrt{a_3}\cosh(k_3x+2\alpha Pk_3t+\xi_0)},\\ \exp(i(Px-(\alpha P^2+2\alpha k_3^2)t)).\tag{39}$$

其中, $\xi_0=\ln\sqrt{a_3}$, $a_3>0$, a_2, P, k_3 为任意常数.

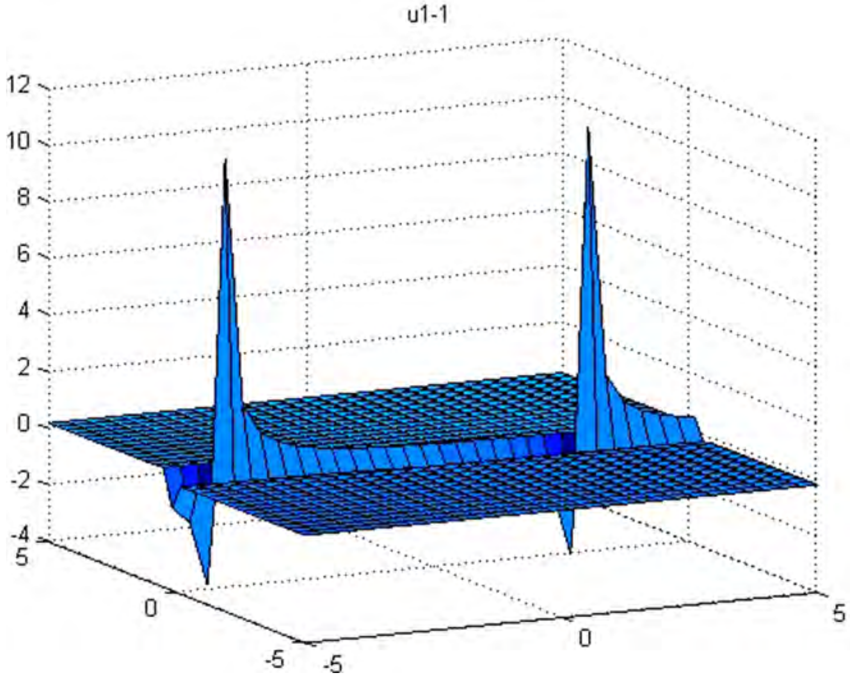

图 1　$a_2 = 10, a_3 = 0.1, k_3 = 0.2, \mathrm{P} = 10, \alpha = 1$ 时 $\left|\sqrt{-\frac{\beta}{2\alpha}u_1}\right|$ 的图形

Fig. 1　figure of $\left|\sqrt{-\frac{\beta}{2\alpha}u_1}\right|$ when $a_2 = 10, a_3 = 0.1, k_3 = 0.2, \mathrm{P} = 10, \alpha = 1$

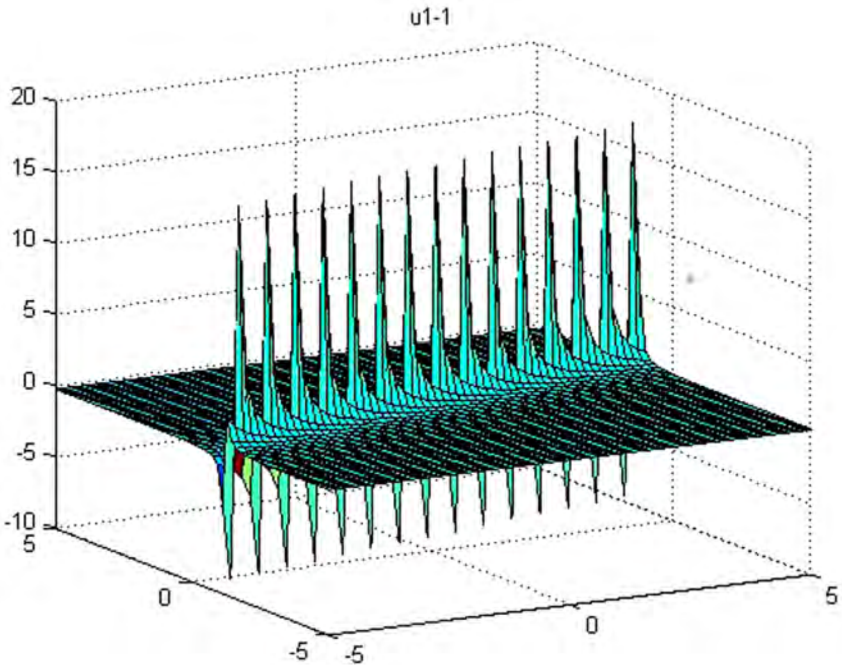

图 2　$a_2 = 20, a_3 = 0.2, k_3 = 0.2, \mathrm{P} = -2, \alpha = 1$ 时 $\left|\sqrt{-\frac{\beta}{2\alpha}u_1}\right|$ 的图形

Fig. 2　figure of $\left|\sqrt{-\frac{\beta}{2\alpha}u_1}\right|$ when $a_2 = 20, a_3 = 0.2, k_3 = 0.2, \mathrm{P} = -2, \alpha = 1$

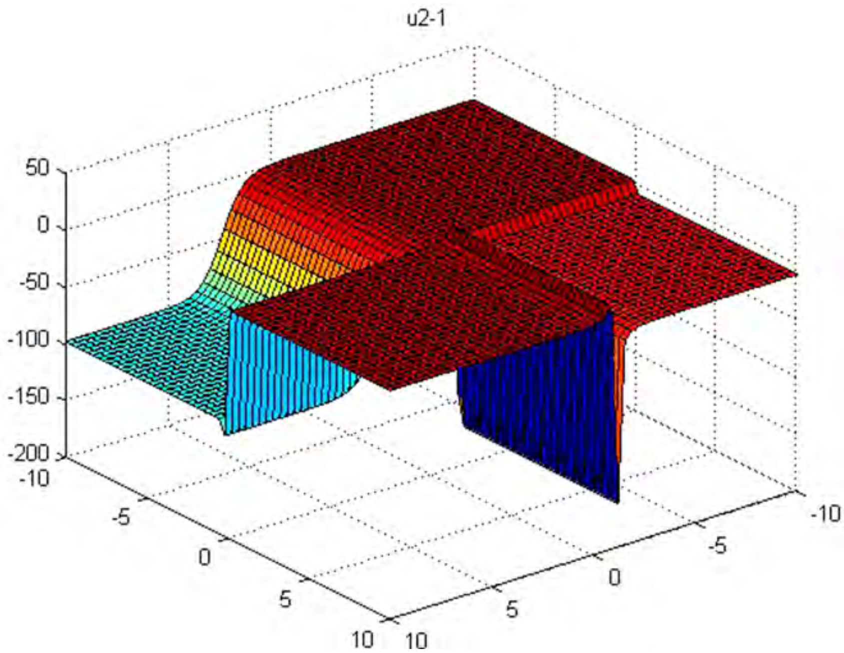

图 3　$a_2=20, a_3=0.1, k_3=1, \mathrm{P}=-10, \alpha=1$ 时 $\left|\sqrt{-\frac{\beta}{2\alpha}u_2}\right|$ 的图形

Fig. 3 figure of $\left|\sqrt{-\frac{\beta}{2\alpha}u_2}\right|$ when $a_2=20, a_3=0.1, k_3=1, \mathrm{P}=-10, \alpha=1$

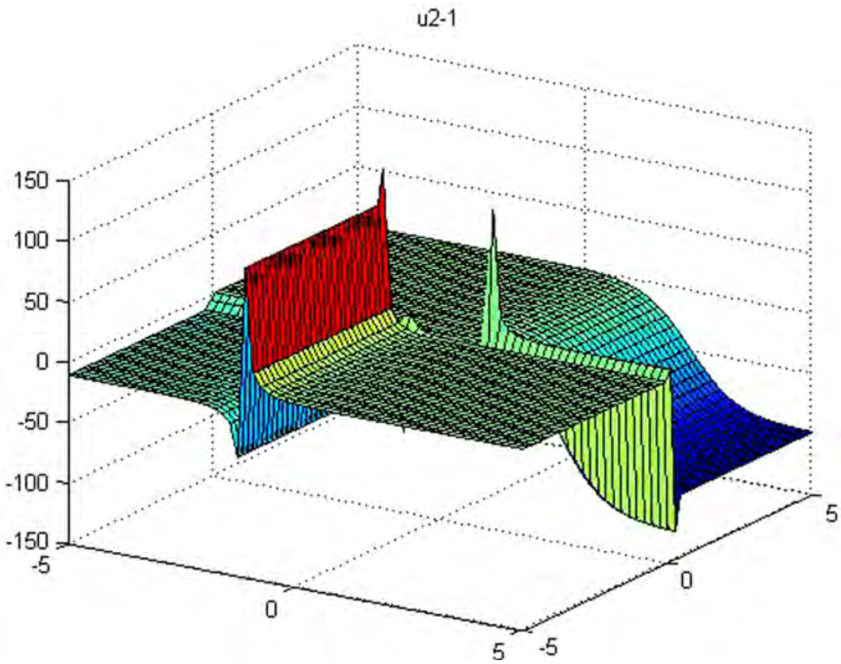

图 4　$a_2=20, a_3=0.1, k_3=1, \mathrm{P}=6, \alpha=1$ 时 $\left|\sqrt{-\frac{\beta}{2\alpha}u_2}\right|$ 的图形

Fig. 3　figure of $\left|\sqrt{-\frac{\beta}{2\alpha}u_2}\right|$ when $a_2=20, a_3=0.1, k_3=1, \mathrm{P}=6, \alpha=1$

②当 $a_3<0$ 时,令 $\xi_0=\ln\sqrt{-a_3}$, 则式(36)可以改写为

$$f(x,t)=a_2\sinh(\pm k_3x\pm 2\alpha\mathrm{P}k_3t)-2\sqrt{-a_3}\sinh(k_3x+2\alpha\mathrm{P}k_3t+\xi_0). \tag{40}$$

把式(40)和式(6)代入式(2)可得方程(1)的双孤波解为

$$u_3(x,t)=\pm\sqrt{-\frac{2\alpha}{\beta}}\frac{a_2k_3\cosh(k_3x+2\alpha\mathrm{P}k_3t)-2\sqrt{-a_3}k_3\cosh(k_3x+2\alpha\mathrm{P}k_3t+\xi_0)}{a_2\sinh(k_3x+2\alpha\mathrm{P}k_3t)-2\sqrt{-a_3}\sinh(k_3x+2\alpha\mathrm{P}k_3t+\xi_0)}\exp(i(\mathrm{P}x-(\alpha\mathrm{P}^2+2\alpha k_3^2)t)).\tag{41}$$

$$u_4(x,t)=\pm\sqrt{-\frac{2\alpha}{\beta}}\frac{a_2k_3\cosh(-k_3x-2\alpha\mathrm{P}k_3t)+2\sqrt{-a_3}k_3\cosh(k_3x+2\alpha\mathrm{P}k_3t+\xi_0)}{a_2\sinh(-k_3x-2\alpha\mathrm{P}k_3t)-2\sqrt{-a_3}\sinh(k_3x+2\alpha\mathrm{P}k_3t+\xi_0)}\exp(i(\mathrm{P}x-(\alpha\mathrm{P}^2+2\alpha k_3^2)t)),\tag{42}$$

其中,$\xi_0=\ln\sqrt{-a_3}$,$a_3<0$,a_2,P,k_3 为任意常数.

研究式子(38),(39),(41)和(42),可以发现 $a_3>0$ 时,方程的解是一种双孤波解,当 $a_3<0$ 时,方程的解则是另一种双孤波解,这说明当平衡点从 $a_3=0$ 的一边穿越到另一边时就会出现时空分岔现象.

情形 II:

$$a_1=0,\ a_2=a_2,\ a_3=-\frac{a_2^2}{4},\ \mathrm{P}=\mathrm{P},\ k_1=k_1,\ k_2=\mp k_3,\ k_3=k_3,$$
$$\Omega=\alpha\mathrm{P}^2+2\alpha k_3^2,\ w_1=w_1,\ w_2=\pm(w_3-4\alpha\mathrm{P}k_3),\ w_3=w_3.\tag{43}$$

把式(43)代入式(9)得

$$f(x,t)=a_2\sinh(\mp k_3x\pm(w_3-4\alpha\mathrm{P}k_3)t)+\exp(-k_3x-w_3t)-\frac{a_2^2}{4}\exp(k_3x+w_3t).\tag{44}$$

令 $\xi_0=\ln\frac{|a_2|}{2}$, 则式(44)可以改写为

$$f(x,t)=a_2\sinh(\mp k_3x\pm(w_3-4\alpha\mathrm{P}k_3)t)-|a_2|\sinh(k_3x+w_3t+\xi_0).\tag{45}$$

把式(45)和式(6)代入式(2)可得方程(1)的双孤波解为

$$u_5(x,t)=\pm\sqrt{-\frac{2\alpha}{\beta}}\frac{a_2k_3\cosh(k_3x-(w_3-4\alpha\mathrm{P}k_3)t)-|a_2|k_3\cosh(k_3x+w_3t+\xi_0)}{a_2\sinh(k_3x-(w_3-4\alpha\mathrm{P}k_3)t)-|a_2|\sinh(k_3x+w_3t+\xi_0)},\exp(i(\mathrm{P}x-(\alpha\mathrm{P}^2+2\alpha k_3^2)t)).\tag{46}$$

$$u_6(x,t)=\pm\sqrt{-\frac{2\alpha}{\beta}}\frac{a_2k_3\cosh(-k_3x+(w_3-4\alpha\mathrm{P}k_3)t)+|a_2|k_3\cosh(k_3x+w_3t+\xi_0)}{a_2\sinh(-k_3x+(w_3-4\alpha\mathrm{P}k_3)t)-|a_2|\sinh(k_3x+w_3t+\xi_0)},\exp(i(\mathrm{P}x-(\alpha\mathrm{P}^2+2\alpha k_3^2)t)).\tag{47}$$

其中,$\xi_0=\ln\frac{|a_2|}{2}$,a_2,P,k_3,w_3 为任意常数.

情形 III:

$$a_1=a_1,\ a_2=0,\ a_3=a_3,\ \mathrm{P}=\mathrm{P},\ k_1=\pm ik_3,\ k_2=k_2,\ k_3=k_3,$$
$$\Omega=\alpha(\mathrm{P}^2+k_3^2),\ w_1=\pm 2i\alpha\mathrm{P}k_3,\ w_2=w_2,\ w_3=2\alpha\mathrm{P}k_3.\tag{48}$$

把式(48)代入式(9)得

$$f(x,t)=a_1\cos(\pm ik_3x\pm 2i\alpha\mathrm{P}k_3t)+\exp(-k_3x-2\alpha\mathrm{P}k_3t)+a_3\exp(k_3x+2\alpha\mathrm{P}k_3t).\tag{49}$$

①当 $a_3>0$ 时,令 $\xi_0=\ln\sqrt{a_3}$, 则式(49)可以改写为

$$f(x,t)=a_1\cos(\pm ik_3x\pm 2i\alpha\mathrm{P}k_3t)+2\sqrt{a_3}\cosh(k_3x+2\alpha\mathrm{P}k_3t+\xi_0).\tag{50}$$

把式(50)和式(6)代入式(2)可得方程(1)的周期双孤波解为

$$u_7(x,t)=\pm\sqrt{-\frac{2\alpha}{\beta}}\frac{\mp ia_1k_3\sin(\pm ik_3x\pm 2i\alpha\mathrm{P}k_3t)+2\sqrt{a_3}k_3\sinh(k_3x+2\alpha\mathrm{P}k_3t+\xi_0)}{a_1\cos(\pm ik_3x\pm 2i\alpha\mathrm{P}k_3t)+2\sqrt{a_3}\cosh(k_3x+2\alpha\mathrm{P}k_3t+\xi_0)}\exp(i(\mathrm{P}x-\alpha(\mathrm{P}^2+2k_3^2)t)),\tag{51}$$

其中, $\xi_0 = \ln\sqrt{a_3}, a_3 > 0, a_1, k_3$ 为任意常数.

②当 $a_3 < 0$ 时,令 $\xi_0 = \ln\sqrt{-a_3}$, 则式(49)可以改写为

$$f(x,t) = a_1\cos(\pm ik_3x \pm 2i\alpha\mathrm{P}k_3t) - 2\sqrt{-a_3}\sinh(k_3x + 2\alpha\mathrm{P}k_3t + \xi_0). \tag{52}$$

把式(52)和式(6)代入式(2)可得方程(1)的周期双孤波解为

$$u_8(x,t) = \pm\sqrt{-\frac{2\alpha}{\beta}}\frac{\mp ia_1k_3\sin(\pm ik_3x \pm 2i\alpha\mathrm{P}k_3t) - 2\sqrt{-a_3}k_3\cosh(k_3x + 2\alpha\mathrm{P}k_3t + \xi_0)}{a_1\cos(\pm ik_3x \pm 2i\alpha\mathrm{P}k_3t) - 2\sqrt{-a_3}\sinh(k_3x + 2\alpha\mathrm{P}k_3t + \xi_0)}$$
$$\exp(i(\mathrm{P}x - \alpha(\mathrm{P}^2 + 2k_3^2)t)), \tag{53}$$

其中, $\xi_0 = \ln\sqrt{-a_3}, a_3 < 0, a_1, k_3$ 为任意常数.

情形 IV:

$$a_1 = a_1,\ a_2 = 0,\ a_3 = \frac{a_1^2}{4},\ \mathrm{P} = \mathrm{P},\ k_1 = \pm ik_3,\ k_2 = k_2,\ k_3 = k_3,$$
$$\Omega = \alpha(\mathrm{P}^2 + k_3^2),\ w_1 = -i(w_3 - 4\alpha\mathrm{P}k_3),\ w_2 = w_2,\ w_3 = w_3. \tag{54}$$

把式(54)代入式(9)得

$$f(x,t) = a_1\cos(\pm ik_3x - i(w_3 - 4\alpha\mathrm{P}k_3)t) + \exp(-k_3x - w_3t) + \frac{a_1^2}{4}\exp(k_3x + w_3t), \tag{55}$$

令 $\xi_0 = \ln\frac{|a_1|}{2}$, 则式(55)可以改写为

$$f(x,t) = a_1\cos(\pm ik_3x - i(w_3 - 4\alpha\mathrm{P}k_3)t) + |a_1|\cosh(k_3x + w_3t + \xi_0), \tag{56}$$

把式(56)和式(6)代入式(2)可得方程(1)的周期双孤波解为

$$u_9(x,t) = \pm\sqrt{-\frac{2\alpha}{\beta}}\frac{\mp ia_1k_3\sin(\pm ik_3x - i(w_3 - 4\alpha\mathrm{P}k_3)t) + |a_1|k_3\sinh(k_3x + w_3t + \xi_0)}{a_1\cos(\pm ik_3x - i(w_3 - 4\alpha\mathrm{P}k_3)t) + |a_1|\cosh(k_3x + w_3t + \xi_0)}$$
$$\exp(i(\mathrm{P}x - \alpha(\mathrm{P}^2 + k_3^2)t)), \tag{57}$$

其中, $\xi_0 = \ln\frac{|a_1|}{2}, a_1, \mathrm{P}, k_3, w_3$ 为任意常数.

情形 V:

$$a_1 = a_1,\ a_2 = a_2,\ a_3 = a_3,\ \mathrm{P} = \mathrm{P},\ k_1 = \pm ik_3,\ k_2 = \pm k_3,\ k_3 = k_3,$$
$$\Omega = \alpha(\mathrm{P}^2 + 2k_3^2),\ w_1 = \pm 2i\alpha\mathrm{P}k_3,\ w_2 = \pm 2\alpha\mathrm{P}k_3,\ w_3 = 2\alpha\mathrm{P}k_3. \tag{58}$$

把式(58)代入式(9)得

$$f(x,t) = a_1\cos(\pm ik_3x \pm 2i\alpha\mathrm{P}k_3t) + a_2\sinh(\pm k_3x \pm 2\alpha\mathrm{P}k_3t) +$$
$$\exp(-k_3x - 2\alpha\mathrm{P}k_3t) + a_3\exp(k_3x + 2\alpha\mathrm{P}k_3t), \tag{59}$$

①当 $a_3 > 0$ 时,令 $\xi_0 = \ln\sqrt{a_3}$, 则式(59)可以改写为

$$f(x,t) = a_1\cos(\pm ik_3x \pm 2i\alpha\mathrm{P}k_3t) + a_2\sinh(\pm k_3x \pm 2\alpha\mathrm{P}k_3t) + 2\sqrt{a_3}\cosh(k_3x + 2\alpha\mathrm{P}k_3t + \xi_0), \tag{60}$$

把式(60)和式(6)代入式(2)可得方程(1)的周期三孤波解为

$$u_{10}(x,t) =$$
$$\pm\sqrt{-\frac{2\alpha}{\beta}}\frac{\mp ia_1k_3\sin(\pm ik_3x \pm 2i\alpha\mathrm{P}k_3t) \pm a_2k_3\cosh(\pm k_3x \pm 2\alpha\mathrm{P}k_3t) + 2\sqrt{a_3}k_3\sinh(k_3x + 2\alpha\mathrm{P}k_3t + \xi_0)}{a_1\cos(\pm ik_3x \pm 2i\alpha\mathrm{P}k_3t) + a_2\sinh(\pm k_3x \pm 2\alpha\mathrm{P}k_3t) + 2\sqrt{a_3}\cosh(k_3x + 2\alpha\mathrm{P}k_3t + \xi_0)}\cdot$$
$$\exp(i(\mathrm{P}x - \alpha(\mathrm{P}^2 + 2k_3^2)t)), \tag{61}$$

其中, $\xi_0 = \ln\sqrt{a_3}, a_3 > 0, a_1, a_2, \mathrm{P}, k_3$ 为任意常数.

②当 $a_3 < 0$ 时,令 $\xi_0 = \ln\sqrt{-a_3}$, 则式(59)可以改写为

$$f(x,t) = a_1\cos(\pm ik_3x \pm 2i\alpha Pk_3t) + a_2\sinh(\pm k_3x \pm 2\alpha Pk_3t) - 2\sqrt{-a_3}\sinh(k_3x + 2\alpha Pk_3t + \xi_0), \quad (62)$$

把式(62)和式(6)代入式(2)可得方程(1)的周期三孤波解为

$$u_{11}(x,t) = \pm\sqrt{-\frac{2\alpha}{\beta}}$$

$$\frac{\mp ia_1k_3\sin(\pm ik_3x \pm 2i\alpha Pk_3t) \pm a_2k_3\cosh(\pm k_3x \pm 2\alpha Pk_3t) - 2\sqrt{-a_3}k_3\cosh(k_3x + 2\alpha Pk_3t + \xi_0)}{a_1\cos(\pm ik_3x \pm 2i\alpha Pk_3t) + a_2\sinh(\pm k_3x \pm 2\alpha Pk_3t) - 2\sqrt{-a_3}\sinh(k_3x + 2\alpha Pk_3t + \xi_0)}\cdot$$

$$\exp(i(Px - \alpha(P^2 + 2k_3^2)t)), \quad (63)$$

其中,$\xi_0 = \ln\sqrt{-a_3}$,$a_3 < 0$,a_1,a_2,P,k_3 为任意常数.

研究式子(61)和(63),可以发现 $a_3 > 0$ 时,方程的解是一种三孤波解,当 $a_3 < 0$ 时,方程的解则是另一种三孤波解,这说明当平衡点从 $a_3 = 0$ 的一边穿越到另一边时同样会出现时空分岔现象.

3 结论

常见的 Hirota 法和其他方法是不能得到同时具有三角函数和双曲函数的解的,本文扩展了 Hirota 法,即将 Hirota 法中的测试函数改用含有三角函数和双曲函数的三波函数来替代,便可以的到这样的解了. 从方程的解中可以看出, Schrödinger 方程孤波的不同形式有不同的时空分岔现象.

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