doi:10.11920/xnmdzk.2018.01.015

马尔可夫跳变时滞系统的无源性分析

李 敏,黄勤珍
(西南民族大学电气信息工程学院,四川 成都 610041)

摘 要:马尔可夫跳变系统作为一种特殊的随机混杂系统,在更好的描述系统结构突然改变的物理系统方面具有优势.因此,在实际应用中,对马尔可夫跳跃系统的研究是一个热门话题.研究了马尔可夫跳变时滞系统的无源性问题.通过构造 Lyapunov-Krasovskii 泛函,导出这类系统满足随机无源性约束的充分条件.最后,给出一个数值例子来验证本文提出理论结果的有效性.

关键词:无源性,马尔可夫跳变系统,Lyapunov 泛函,时滞
中图分类号:O231;TP13 **文献标志码**:A **文章编号**:2095-4271(2018)01-0097-07

Passivity analysis for Markovian jump systems with time-varying delays

LI Min,HUANG Qin-zhen
(School of Electrical and Information Engineering,Southwest Minzu University,Chengdu 610041,P. R. C.)

Abstract:Markovian jump systems,as a special kind of stochastic hybrid systems,have the advantages for better representing physical systems with the abrupt changes of their structures. So,the research of Markovian jump system is a hot topic in practical applications. This paper studies the problem of passivity analysis for Markovian jump systems with time-varying delays. By constructing a Lyapunov-Krasovskii functional, sufficient criteria of stochastic passivity for Markovian jump systems are derived. Finally,a numerical example is given to verify the effectiveness of the proposed results.

Key words:passivity analysis;Markovian jump system;Lyapunov functional;time delay

马尔可夫跳变系统是一类特殊的随机混杂系统.马尔可夫跳变往往来源于系统在运行过程中所受到的环境突变、内部子系统连接方式突然改变、系统部件损坏等随机因素干扰[1-2].因此,研究马尔可夫跳变系统为解决工程控制问题提供了理论基础[3-4].众所周知,时滞广泛存在于各种实际系统中,然而,它的存在会使系统不稳定或性能遭到破坏[5].因此,研究马尔可夫时滞跳变系统具有实际的意义.

许多实际系统通过考虑无源性问题可以有效地抑制外界噪声干扰[6-7].近年来,系统无源性研究成为了一个重要的热点问题[8-9],吸引了众多学者的关注[10-12].文献[1]给出时滞马尔可夫跳变系统的随机无源性定义.文献[13]分析了线性时滞系统的时滞相关无源控制问题.文献[14]讨论了时变时滞的神经网络无源性问题.

本文研究了具有不确定性矩阵的马尔可夫跳变时滞系统的无源性问题.文章其余部分结构如下:第 1 节介绍本文研究的马尔可夫跳变时滞系统和推导所需的定义与引理.第 2 节得到满足系统无源性约束的充分判据.第 3 节给出一个数值例子验证所得理论结果的有效性与可行性.第 4 节总结全文.

符号:$\mathbb{R}^n$和$\mathbb{R}^{n\times m}$分别定义为n维欧氏空间和$n\times m$实矩阵.I为合适维数的单位阵.当0表示一个矩阵时,它代表$n\times n$维零矩阵或能很容易从上下文中看出维数.A^{T} 和 A^{-1} 分别为矩阵 A 的转置矩阵和逆矩阵.

收稿日期:2017-11-13
通信作者:黄勤珍(1965-),女,福建人,教授,研究方向:自动化控制.E-mail:13548002166@139.com
基金项目:国家自然科学基金(61673016);四川省教育厅创新团队(15TD0050)

$A>0(<0)$表示矩阵A是正定(负定)矩阵."$*$"表示对称矩阵的对称块. $L_2[0,\infty)$表示在$[0,\infty)$上是平方可积的向量函数空间.

1 预备知识

给定一个完备概率空间$\{\Omega,\mathcal{F},\mathcal{P}\}$,考虑如下马尔可夫跳变系统:

$$\begin{cases} \dot{x}(t) = (A_{r(t)}+\Delta A_{r(t)}(t))x(t)+(B_{r(t)}+\Delta B_{r(t)}(t))x(t-\tau(t))+C_{r(t)}\omega(t), \\ z(t) = \tilde{A}_{r(t)}x(t)+\tilde{B}_{r(t)}x(t-\tau(t))+\tilde{C}_{r(t)}\omega(t), \\ x(t) = \varphi(t), \quad t\in[-h_2,\ 0]. \end{cases} \tag{1}$$

其中,$x(t)\in\mathbb{R}^n$是状态变量,$z(t)\in\mathbb{R}^m$是测量输出信号,$\omega(t)\in\mathbb{R}^p$是外部扰动输入且属于空间$L_2[0,\infty)$. $\{r(t),t\geq 0\}$是在有限集合$\mathbb{S}=\{1,\ldots,s\}$上取值的连续时间、离散状态马尔可夫过程. 转移率矩阵$\Pi=\{\pi_{ij}\}$满足

$$P_r\{r(t+\Delta)=j|r(t)=i\}=\begin{cases} \pi_{ij}\Delta+o(\Delta), & j\neq i \\ 1+\pi_{ii}\Delta+o(\Delta), & j=i \end{cases}$$

式中,$\lim_{\Delta\longrightarrow 0}\frac{o(\Delta)}{\Delta}(\Delta>0)$,$\pi_{ij}\geq 0(j\neq i)$是模态$i$到模态$j$的转移率,且满足$\pi_{ii}=-\sum_{j=1,j\neq i}^{s}\pi_{ij}$.

$A_i,B_i,C_i,\tilde{A}_i,\tilde{B}_i,\tilde{C}_i$是具有合适维数的常矩阵,$\Delta A_i(t),\Delta B_i(t)$是不确定矩阵,满足条件$[\Delta A_i(t),\Delta B_i(t)]=G_{1i}F_{1i}(t)[N_{1i},N_{2i}]$,其中$G_{1i},N_{1i},N_{2i}$是已知常矩阵,$F_{1i}(t)$满足$F_{1i}^{\mathrm{T}}(t)F_{1i}(t)\leq I$. 时变时滞$\tau(t)$满足$0\leq\underline{\tau}\leq\tau(t)\leq\bar{\tau},\ \dot{\tau}(t)\leq d$,其中$\underline{\tau},\ \bar{\tau},\ d$是常数.

因此,系统(1)可简化为:

$$\begin{cases} \dot{x}(t) = (A_i+\Delta A_i(t))x(t)+(B_i+\Delta B_i(t))x(t-\tau(t))+C_i\omega(t), \\ z(t) = \tilde{A}_ix(t)+\tilde{B}_ix(t-\tau(t))+\tilde{C}_i\omega(t), \\ x(t) = \varphi(t), \quad t\in[-h_2,\ 0]. \end{cases} \tag{2}$$

接下来,我们将介绍一些有用的定义和引理.

定义1[1] 如果存在一个正标量$\gamma>0$,在零初始条件下,对任意的非零量$\omega(t)\in L_2[0,\infty)$,使得不等式$\mathbb{E}\left\{2\int_{t_0}^{t^*}z^{\mathrm{T}}(t)\omega(t)\mathrm{d}t\right\}\geq-\gamma\int_{t_0}^{t^*}\omega^{\mathrm{T}}(t)\omega(t)\mathrm{d}t,\forall t^*\geq t_0$成立,则系统(2)是鲁棒随机无源的.

引理1[15] 令$n\times n$维矩阵$R>0$,常数$\tau>0$,向量值函数$\dot{x}:[-\tau,0]\longrightarrow\mathbb{R}^n$使得下面的不等式成立:

$$-\tau\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\,\mathrm{d}s\leq\begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau)\end{bmatrix}\begin{bmatrix} -R & R \\ * & -R\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\tau)\end{bmatrix},$$

$$-\frac{\tau^2}{2}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\,\mathrm{d}s\mathrm{d}\theta\leq\begin{bmatrix} \tau x^{\mathrm{T}}(t) & \int_{t-\tau}^{t}x^{\mathrm{T}}(s)\,\mathrm{d}s\end{bmatrix}\begin{bmatrix} -R & R \\ * & -R\end{bmatrix}\begin{bmatrix} \tau x(t) \\ \int_{t-\tau}^{t}x(s)\,\mathrm{d}s\end{bmatrix}.$$

引理2[16] 令矩阵$V=V^{\mathrm{T}},G,E,F(t)$为合适维数的实矩阵,且$F(t)$满足$F^{\mathrm{T}}(t)F(t)\leq I$. 则不等式$V+GF(t)E+E^{\mathrm{T}}F^{\mathrm{T}}(t)G^{\mathrm{T}}<0$成立当且仅当存在一个正的常数$\varepsilon>0$使得$V+\varepsilon GG^{\mathrm{T}}+\varepsilon^{-1}E^{\mathrm{T}}E<0$成立.

2 主要定理

本节主要推导保证系统(2)具有随机无源性的充分条件.

定理1 若存在对称正定矩阵$P_i,Q_i\in\mathbb{R}^{n\times n}(i\in\mathbb{S}),Q,T,Z\in\mathbb{R}^{n\times n}$,常数$\varepsilon>0,\gamma>0$使得如下线性矩阵不等式成立:

$$\begin{bmatrix} \Theta_i & G_i & N_i^{\mathrm{T}} \\ * & -\varepsilon^{-1}I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, i \in \mathbb{S} \tag{3}$$

$$\sum_{j=1}^{s} \pi_{ij} Q_j - Q \leq 0, i \in \mathbb{S} \tag{4}$$

其中

$$\Theta_i = \begin{bmatrix} \Sigma_i & \Phi_i \\ * & -(2\tilde{C}_i + \gamma I) \end{bmatrix}, G_i^{\mathrm{T}} = \begin{bmatrix} G_{1i}^{\mathrm{T}} M_1 & G_{1i}^{\mathrm{T}} M_2 & G_{1i}^{\mathrm{T}} M_3 & G_{1i}^{\mathrm{T}} M_4 & G_{1i}^{\mathrm{T}} M_5 & 0 \end{bmatrix},$$

$$N_i = [N_{1i} \ \ 0 \ \ N_{2i} \ \ 0 \ \ 0 \ \ 0], \Phi_i^{\mathrm{T}} = \begin{bmatrix} C_i^{\mathrm{T}} M_1 - \tilde{A}_i & C_i^{\mathrm{T}} M_2 & C_i^{\mathrm{T}} M_3 - \tilde{B}_i & C_i^{\mathrm{T}} M_4 & C_i^{\mathrm{T}} M_5 \end{bmatrix},$$

$$\Sigma_i = \begin{bmatrix} \Psi_i & P_i + A_i^{\mathrm{T}} M_2 - M_1^{\mathrm{T}} & A_i^{\mathrm{T}} M_3 + M_1^{\mathrm{T}} B_i & Z + A_i^{\mathrm{T}} M_4 & \underline{\tau} T + A_i^{\mathrm{T}} M_5 \\ * & \underline{\tau}^2 Z + \frac{\underline{\tau}^4}{4} T - M_2^{\mathrm{T}} - M_2 & M_2^{\mathrm{T}} B_i - M_3 & -M_4 & -M_5 \\ * & * & -(1-d)Q_i + M_3^{\mathrm{T}} B_i + B_i^{\mathrm{T}} M_3 & B_i^{\mathrm{T}} M_4 & B_i^{\mathrm{T}} M_5 \\ * & * & * & -Z & 0 \\ * & * & * & * & -T \end{bmatrix},$$

$$\Psi_i = Q_i + \sum_{j=1}^{s} \pi_{ij} P_j + \bar{\tau} Q - Z - \underline{\tau}^2 Z + M_1^{\mathrm{T}} A_i + A_i^{\mathrm{T}} M_1,$$

则系统(2)是随机无源的.

证明　考虑如下的 Lyapunov 函数:

$$V(x_t, t, i) = V_1(x_t, t, i) + V_2(x_t, t, i) + V_3(x_t, t, i), \tag{5}$$

其中

$$V_1(x_t, t, i) = x^{\mathrm{T}}(t) P_i x(t) + \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s) Q_i x(s) \mathrm{d}s,$$

$$V_2(x_t, t, i) = \int_{-\bar{\tau}}^{0} \int_{t+\beta}^{t} x^{\mathrm{T}}(s) Q x(s) \mathrm{d}s \, \mathrm{d}\beta + \underline{\tau} \int_{-\underline{\tau}}^{0} \int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(s) Z \dot{x}(s) \mathrm{d}s \, \mathrm{d}\beta,$$

$$V_3(x_t, t, i) = \frac{\underline{\tau}^2}{2} \int_{-\underline{\tau}}^{0} \int_{\beta}^{0} \int_{t+\lambda}^{t} \dot{x}^{\mathrm{T}}(s) T \dot{x}(s) \mathrm{d}s \, \mathrm{d}\lambda \mathrm{d}\beta,$$

随机过程$\{x_t, r(t), t \geq 0\}$在点$\{x_t, t, i\}$处的弱无穷小算子$\mathfrak{L}(\cdot)$运算如下:

$$\mathfrak{L}V(x_t, t, i) = \frac{\partial V}{\partial t} + \dot{x}(t) \frac{\partial V}{\partial x}\Big|_{r(t)=i} + \sum_{j=1}^{s} \pi_{ij} V(x_t, t, j),$$

对(5)式计算可得

$$\begin{aligned} \mathfrak{L}V_1(x_t, t, i) \leq & 2x^{\mathrm{T}}(t) P_i \dot{x}(t) + x^{\mathrm{T}}(t) \sum_{j=1}^{s} \pi_{ij} P_j x(t) + x^{\mathrm{T}}(t) Q_i x(t) - \\ & (1-d) x^{\mathrm{T}}(t-\tau(t)) Q_i x(t-\tau(t)) + \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s) \sum_{j=1}^{s} \pi_{ij} Q_j x(s) \mathrm{d}s. \end{aligned} \tag{6}$$

$$\begin{aligned}\mathfrak{L}V_2(x_t,t,i) =&\bar{\tau}x^{\mathrm{T}}(t)Qx(t) - \int_{t-\bar{\tau}}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s + \dot{x}^{\mathrm{T}}(t)\left(\underline{\tau}^2 Z\right)\dot{x}(t) - \\ &\underline{\tau}\int_{t-\underline{\tau}}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s \leq \\ &\bar{\tau}x^{\mathrm{T}}(t)Qx(t) - \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s + \dot{x}^{\mathrm{T}}(t)\left(\underline{\tau}^2 Z\right)\dot{x}(t) - \\ &\underline{\tau}\int_{t-\underline{\tau}}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s\,,\end{aligned} \tag{7}$$

$$\mathfrak{L}V_3(x_t,t,i) \leq \frac{\underline{\tau}^4}{4}\dot{x}^{\mathrm{T}}(s)T\dot{x}(s) - \frac{\underline{\tau}^2}{2}\int_{-\underline{\tau}}^{0}\int_{t+\beta}^{t}\dot{x}^{\mathrm{T}}(s)T\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\,, \tag{8}$$

由引理 1,可得

$$\begin{aligned}-\underline{\tau}\int_{t-\underline{\tau}}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s \leq \begin{bmatrix} x(t) \\ x(t-\underline{\tau})\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -Z & Z \\ * & -Z\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\underline{\tau})\end{bmatrix} - \\ \frac{\underline{\tau}^2}{2}\int_{-\underline{\tau}}^{0}\int_{t+\beta}^{t}\dot{x}^{\mathrm{T}}(s)T\dot{x}(s)\mathrm{d}s\mathrm{d}\beta \leq \begin{bmatrix} \underline{\tau}x(t) \\ \int_{t-\underline{\tau}}^{t} x(s)\mathrm{d}s\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -T & T \\ * & -T\end{bmatrix}\begin{bmatrix} \underline{\tau}x(t) \\ \int_{t-\underline{\tau}}^{t} x(s)\mathrm{d}s\end{bmatrix}\,,\end{aligned} \tag{9}$$

由(5)式到(9)式可知

$$\mathfrak{L}V(x_t,t,i) \leq \xi^{\mathrm{T}}(t)\Omega_i\xi(t) + \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s)(\sum_{j=1}^{s}\pi_{ij}Q_j - Q)x(s)\mathrm{d}s\,, \tag{10}$$

其中

$$\Omega_i = \begin{bmatrix} Q_i + \sum_{j=1}^{s}\pi_{ij}P_j + \bar{\tau}Q - Z - \underline{\tau}^2 Z & P_i & 0 & Z & \underline{\tau}T \\ * & \underline{\tau}^2 Z + \frac{\underline{\tau}^4}{4}T & 0 & 0 & 0 \\ * & * & -(1-d)Q_i & 0 & 0 \\ * & * & * & -Z & 0 \\ * & * & * & * & -T \end{bmatrix},$$

$$\xi(t) = \left[x^{\mathrm{T}}(t), \dot{x}^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\tau(t)), x^{\mathrm{T}}(t-\underline{\tau}), \int_{t-\underline{\tau}}^{t} x^{\mathrm{T}}(s)\mathrm{d}s\right]^{\mathrm{T}}.$$

对于任意矩阵 $M^{\mathrm{T}} = \begin{bmatrix} M_1 & M_2 & M_3 & M_4 & M_5\end{bmatrix}$,有如下等式成立

$$\begin{aligned}0 =&2\xi^{\mathrm{T}}(t)M\left((A_i+\Delta A_i(t))x(t) + (B_i+\Delta B_i(t))x\left(t-\tau(t)\right) + C_i\omega(t) - \dot{x}(t)\right) \\ =&2\xi^{\mathrm{T}}(t)M\begin{bmatrix} A_i & -I & B_i & 0 & 0\end{bmatrix}\xi(t) + \\ &2\xi^{\mathrm{T}}(t)M\begin{bmatrix} \Delta A_i(t) & 0 & \Delta B_i(t) & 0 & 0\end{bmatrix}\xi(t) + 2\xi^{\mathrm{T}}(t)MC_i\omega(t) \\ =&\xi^{\mathrm{T}}(t)(\Gamma_i + \Delta\Gamma_i)\xi_1(t) + 2\xi^{\mathrm{T}}(t)MC_i\omega(t)\,.\end{aligned} \tag{11}$$

其中

$$\Gamma_i = \begin{bmatrix} M_1^{\mathrm{T}}A_i + A_i^{\mathrm{T}}M_1 & A_i^{\mathrm{T}}M_2 - M_1^{\mathrm{T}} & A_i^{\mathrm{T}}M_3 + M_1^{\mathrm{T}}B_i & A_i^{\mathrm{T}}M_4 & A_i^{\mathrm{T}}M_5 \\ * & -M_2^{\mathrm{T}} - M_2 & M_2^{\mathrm{T}}B_i - M_3 & -M_4 & -M_5 \\ * & * & M_3^{\mathrm{T}}B_i + B_i^{\mathrm{T}}M_3 & B_i^{\mathrm{T}}M_4 & B_i^{\mathrm{T}}M_5 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

$$\Delta\Gamma_i=\begin{bmatrix} M_1^{\mathrm T}\Delta A_i(t)+\Delta A_i^{\mathrm T}(t)M_1 & \Delta A_i^{\mathrm T}(t)M_2 & \Delta A_i^{\mathrm T}(t)M_3+M_1^{\mathrm T}\Delta B_i(t) & \Delta A_i^{\mathrm T}(t)M_4 & \Delta A_i^{\mathrm T}(t)M_5 \\ * & 0 & M_1^{\mathrm T}\Delta B_i(t) & 0 & 0 \\ * & * & \Delta B_i^{\mathrm T}(t)M_3+M_3^{\mathrm T}\Delta B_i(t) & \Delta B_i^{\mathrm T}(t)M_4 & \Delta B_i^{\mathrm T}(t)M_5 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

(10)式和(11)式表明

$$\mathfrak{L}V(x_t,t,i)\le\xi^{\mathrm T}(t)(\Omega_i+\Gamma_i+\Delta\Gamma_i)\xi(t)+\int_{t-\tau(t)}^{t}x^{\mathrm T}(s)(\sum_{j=1}^{s}\pi_{ij}Q_j-Q)x(s)\mathrm{d}s+2\xi^{\mathrm T}(t)MC_i\omega(t)$$

由上式易知

$$\mathbb{E}\left\{\mathfrak{L}V(x_t,t,i)-2z^{\mathrm T}(t)\omega(t)-\gamma\omega^{\mathrm T}(t)\omega(t)\right\}\le\mathbb{E}\left\{\eta^{\mathrm T}(t)\Lambda_i\eta(t)+\int_{t-l(t)}^{t}x^{\mathrm T}(s)(\sum_{j=1}^{s}\pi_{ij}Q_j-Q)x(s)\mathrm{d}s\right\},$$

其中，$\eta(t)=[\xi^{\mathrm T}(t),\omega^{\mathrm T}(t)]^{\mathrm T}$，$\Phi_i^{\mathrm T}=\begin{bmatrix}C_i^{\mathrm T}M_1-\tilde A_i & C_i^{\mathrm T}M_2 & C_i^{\mathrm T}M_3-\tilde B_i & C_i^{\mathrm T}M_4 & C_i^{\mathrm T}M_5\end{bmatrix}$，

$$\Lambda_i=\begin{bmatrix}\Sigma_i+\Delta\Gamma_i & \Phi_i \\ * & -(2\tilde C_i+\gamma I)\end{bmatrix},$$ Σ_i在定理 1 里有定义．

由引理 2，可知

$$\Lambda_i=\begin{bmatrix}\Sigma_i & \Phi_i \\ * & -(2\tilde C_i+\gamma I)\end{bmatrix}+\begin{bmatrix}\Delta\Gamma_i & 0 \\ * & 0\end{bmatrix}=$$
$$\begin{bmatrix}\Sigma_i & \Phi_i \\ * & -(2\tilde C_i+\gamma I)\end{bmatrix}+G_iF_i(t)N_i+N_i^{\mathrm T}F_i^{\mathrm T}(t)G_i^{\mathrm T}\le$$
$$\begin{bmatrix}\Sigma_i & \Phi_i \\ * & -(2\tilde C_i+\gamma I)\end{bmatrix}+\varepsilon G_iG_i^{\mathrm T}+\varepsilon^{-1}N_i^{\mathrm T}N_i\,.$$

其中，$G_i^{\mathrm T}=\begin{bmatrix}G_{1i}^{\mathrm T}M_1 & G_{1i}^{\mathrm T}M_2 & G_{1i}^{\mathrm T}M_3 & G_{1i}^{\mathrm T}M_4 & G_{1i}^{\mathrm T}M_5 & 0\end{bmatrix}$，$N_i=[N_{1i}\ \ 0\ \ N_{2i}\ \ 0\ \ 0\ \ 0]$，

根据 Schur 补，由(3)式和引理 2 可知

$$\begin{bmatrix}\Sigma_i & \Phi_i \\ * & -(2\tilde C_i+\gamma I)\end{bmatrix}+\varepsilon G_iG_i^{\mathrm T}+\varepsilon^{-1}N_i^{\mathrm T}N_i\le 0, \tag{12}$$

由(4)式有

$$\mathbb{E}\left\{\mathfrak{L}V(x_t,t,i)-2z^{\mathrm T}(t)\omega(t)-\gamma\omega^{\mathrm T}(t)\omega(t)\right\}\le 0, \tag{13}$$

因此，对(13)式从0到T积分可得

$$2\mathbb{E}\int_0^T z^{\mathrm T}(t)\omega(t)\mathrm{d}t\ge\mathbb{E}V(x_t,t,i)-\mathbb{E}V(x_0,0,i)-\gamma\mathbb{E}\int_0^T\omega^{\mathrm T}(t)\omega(t)\mathrm{d}t\ge-\gamma\mathbb{E}\int_0^T\omega^{\mathrm T}(t)\omega(t)\mathrm{d}t$$ 根据定理 1，可以得到系统(2)是随机无源的，证毕.

3　数值例子

这部分用一个例子来验证所提出的主要结果．

例 1　给定一个完备概率空间$\{\Omega,\mathcal{F},\mathcal{P}\}$，考虑一个两模态的马尔可夫跳变系统(2)，具体参数设置如下：

$$A_1=\begin{bmatrix}-4 & 0.2\\ -0.2 & -1\end{bmatrix}, A_2=\begin{bmatrix}-2.4 & 0.5\\ 0.4 & -1.4\end{bmatrix}, B_1=\begin{bmatrix}0.4 & 0\\ 0.1 & -0.1\end{bmatrix}, B_2=\begin{bmatrix}0.6 & -0.5\\ 0.1 & -0.1\end{bmatrix},$$

$$\tilde{A}_1=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}, \tilde{A}_2=\begin{bmatrix}-2 & 0\\ 0 & -3\end{bmatrix}, \tilde{B}_1=\begin{bmatrix}0 & 0.8\\ 0 & -1.68\end{bmatrix}, \tilde{B}_2=\begin{bmatrix}0 & 0.8\\ 0 & -0.32\end{bmatrix},$$

$$\tilde{C}_1=\begin{bmatrix}0 & 0.1\\ 0.1 & 0\end{bmatrix}, \tilde{C}_2=\begin{bmatrix}0 & 0.1\\ 0.1 & 0\end{bmatrix}, C_1=\begin{bmatrix}0 & 0.1\\ 0.1 & 0\end{bmatrix}, C_2=\begin{bmatrix}0 & 0.1\\ 0.1 & 0\end{bmatrix},$$

$$G_{11}=\begin{bmatrix}-0.3 & 0.1\\ 0 & 0.1\end{bmatrix}, G_{12}=\begin{bmatrix}-0.2 & 0.1\\ 0 & 0.1\end{bmatrix}, N_{11}=\begin{bmatrix}-0.3 & 0.01\\ 0 & 0.2\end{bmatrix}, N_{12}=\begin{bmatrix}-0.2 & 0.1\\ 0 & 0.1\end{bmatrix},$$

$$N_{21}=\begin{bmatrix}-0.1 & 0.01\\ 0 & 0.2\end{bmatrix}, N_{22}=\begin{bmatrix}-0.2 & 0.1\\ 0 & 0.1\end{bmatrix}, \Pi=\begin{bmatrix}-0.1 & 0.1\\ 0.8 & -0.8\end{bmatrix}.$$

给定$\underline{\tau}=0,\ \gamma=0.24,\ \varepsilon=0.2$,根据定理 1,对于不同的时滞导数取值$d$,表 1 为满足系统(2)无源性的最大时滞上界表.

表 1 最大时滞上界 T

Table 1 Upper bounds of T for different values of d

方法	$d=0$	$d=0.1$	$d=0.3$
定理 1	4.9052	4.0321	1.6885

给定初始模态为 1,$\varphi(t)=[0.02\ \ 0.02]^{\mathrm{T}}$,$\tau(t)=1+0.3\sin^2(t)$,$\omega(t)=\exp(-0.2t)\sin(t)[1\ \ 1]^{\mathrm{T}}$,图 1 为输出信号图,图 2 为切换信号图. 从图中很容易看出,系统(2)是随机无源的.

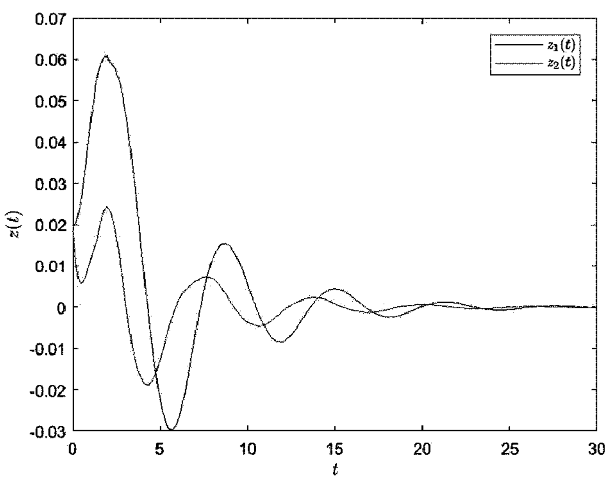

图 1 输出信号图

Fig. 1 Output signal of system

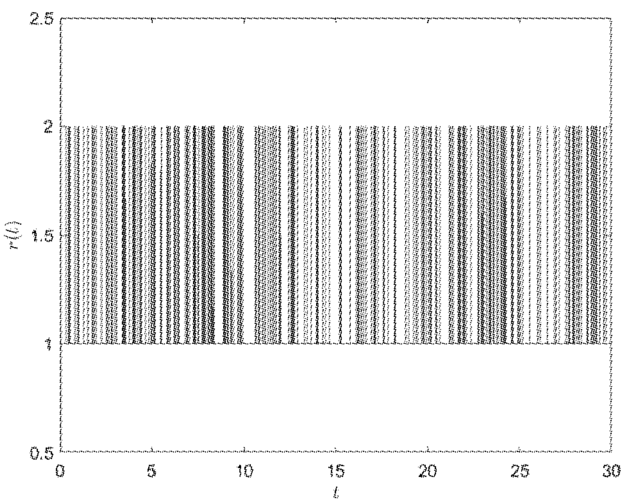

图 2 切换信号图

Fig. 2 The switching signals

4 结论

本文研究了马尔可夫跳变系统的无源性问题,其中,转移概率是已知的. 通过构建 Lyapunov 泛函,得到马尔可夫跳变时滞系统的随机无源性充分判据. 在实际系统中,转移概率是变化或未知的,研究不确定马尔可夫跳变系统具有一定的挑战性. 因此,在未来的工作中将进一步对不确定马尔可夫时滞系统展开研究.

参考文献

[1]WU Y Q,SU H,LU R,et al. Passivity-based non-fragile control for Markovian jump systems with aperiodic sampling[J]. Systems & Control Letters,2015,84:35-43.

[2] SONG J,NIU Y,ZOU Y. Asynchronous output feedback control of time-varying Markovian jump systems within a finite-time interval[J]. Journal of the Franklin Institute,2017,354(15):6747-6765,.

[3]RAKKIYAPPAN R,SASIREKHA R,LAKSHMANAN S,et al. Synchronization of discrete-time Markovian jump complex dynamical networks with random delays via non-fragile control[J]. Journal of the Franklin Institute,2016,353(16):4300-4329.

[4]FEI Z,GAO H,SHI P . New results on stabilization of Markovian jump systems with time delay[J]. Automatica,2009,45(10):2300-2306.

[5]LIU X,ZHONG S,ZHAO Q. Dynamics of delayed switched nonlinear systems with applications to cascade systems[J]. Automatica,2018,87:251-257.

[6]YUE D,LAM J. Non-fragile guaranteed cost control for uncertain descriptor systems with time-varying state and input delays[J]. Optimal Control Applications & Methods,2005,26(2):85-105,.

[7]YANG G H,WANG J L. Non-fragile H∞ control for linear systems with multiplicative controller gain variations[J]. Automatica,2001,37(5):727-737.

[8]SHEN H,XU S,LU J,et al. Passivity-based control for uncertain stochastic jumping systems with mode-dependent round-trip time delays[J]. Journal of the Franklin Institute,2012,349(5):1665-1680.

[9]HE S,LIU F. Optimal finite-time passive controller design for uncertain nonlinear Markovian jumping systems[J]. Journal of the Franklin Institute,2014,351(7):3782-3796.

[10]WANG B,ZHU Q. Stability analysis of Markov switched stochastic differential equations with both stable and unstable subsystems[J]. Systems & Control Letters,2017,105:55-61.

[11]CHEN G,XIA J,ZHUANG G. Delay-dependent stability and dissipativity analysis of generalized neural networks with Markovian jump parameters and two delay components[J]. Journal of the Franklin Institute,2016,353(9):2137-2158.

[12]MA C. Non-fragile mixed H∞ and passive synchronization of Markov jump neural networks with mixed time-varying delays and randomly occurring controller gain fluctuation[J]. PLoS One,2017,12(4):e0175676.

[13]XIA Z,LI J,LI J. Passivity-based resilient adaptive control for fuzzy stochastic delay systems with Markovian switching[J]. Journal of the Franklin Institute,2014,351(7):3818-3836.

[14]ZHANG Z,MOU S,LAM J,et al. New passivity criteria for neural networks with time-varying delay[J]. Neural Networks,2009,22(7):864-868.

[15]HAN Q L. A new delay-dependent stability criterion for linear neutral systems with norm-bounded uncertainties in all system matrices[J]. International Journal of Systems Science,2005,36(8):469-475.

[16]XIE L. Output feedback H∞ control of systems with parameter uncertainty[J]. International Journal of Control,1996,63(4):741-750.

(责任编辑:张阳,付强,李建忠,罗敏;英文编辑:周序林)