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关于对称共轭点的亚纯双向单叶倒星象函数类的系数估计

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摘　要:在单复变函数几何理论的研究中,构造函数类及研究它的几何性质是重要的研究课题. 而在几何性质的研究中,对于系数的估计具有重要的作用. 国内外许多学者对于单叶函数类和多叶函数类都进行了较为深入的研究. 而对于亚纯函数类尤其是倒结构的亚纯函数类的研究却很少. 引入了一类关于对称共轭点的亚纯双向单叶倒星象函数类,得到了该函数类的积分表达式和系数估计. 特别地,得到了 Fekete-Szegö 问题的精确估计.

关键词:双向单叶;亚纯;对称共轭点;积分表达式;系数估计;Fekete-Szegö

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Coefficient estimate of some class of meromorphic bi-valent reciprocal starlike functions with respect to symmetric conjugate points

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Abstract: In the study of the geometry theory of functions of one complex variable, it is an important research topic to construct the class of function and study its geometric properties. In the study of the geometric nature, the estimate of coefficient plays an important role. There are a lot of research for univalent function and multivalent function by many scholars at home and abroad. However, there is very little research on the meromorphic function class. In the present paper, we introduced some class of meromorphic bi-univalent reciprocal starlike functions with respect to symmetric conjugate points and obtained the integral representation and coefficient estimate. Specially, we obtained the sharp estimate of Fekete-Szegö problem.

Key words: bi-univalent; meromorphic; symmetric conjugate point; integral representation; coefficient estimate; Fekete-Szegö

1　引言

令 A 表示单位圆盘 $U=\{z\in C:|z|<1\}$ 内解析且具有如下形式

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n\ ,$$

的函数类.

设 Σ 表示去心单位圆盘 $U^*=\{z\in C:|z|<1\}=U\backslash\{0\}$ 内解析且具有如下形式的亚单纯叶函数类

$$f(z)=z^{-1}+\sum_{k=1}^{\infty}a_kz^k\ . \tag{1}$$

对于每一个函数 $f(z)\in\Sigma$,具有逆函数 f^{-1},定义为

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$$f^{-1}(f(z)) = z \quad (z \in U^*)$$

及

$$f(f^{-1}(\omega)) = \omega \quad \left(|\omega| < r_0(f);r_0(f) \geq \frac{1}{4}\right).$$

对于具有(1)形式的函数$f(z)$的逆函数f^{-1}具有如下形式,

$$f^{-1}(\omega) = \omega^{-1} - a_1\omega - a_2\omega^2 + \cdots. \tag{2}$$

若函数f和f^{-1}都在U^*内单叶,则称函数$f(z) \in \Sigma$在U^*内亚纯双向单叶. 近来,许多学者对亚纯双向单叶函数进行了研究,详见文献[1-7].

令P表示在U内解析且具有如下形式

$$p(z) = 1 + \sum_{k=1}^{\infty} p_k z^k,$$

的函数$p(z)$的全体,且$\mathrm{Re}\,p(z) > 0$.

设函数$u(z)$和$v(z)$在A中解析,若存在一个 Schwarz 函数ϖ,在U内满足$\varpi(0)=0$和$|\varpi(z)|<1$,使得$u(z) = v(\varpi(z))(z \in U)$,则称函数$u(z)$从属于$v(z)$,记作$u(z) \prec v(z)$. 另外,若$v$在$U$内单叶,则$u(z) \prec v(z)$等价于

$$u(0) = v(0) \text{ 和 } u(U) \subset v(U).$$

函数$f(z) \in A$属于函数类$S^*(\varphi)$,如果满足如下条件

$$\frac{zf'(z)}{f(z)} \prec \varphi(z),$$

其中$\varphi(z) \in P$. 函数类$S^*(\varphi)$和相应的凸函数类$K(\varphi)$由 Ma 和 Minda 定义[8].

1959 年,Sakaguchi[9]引入关于对称点的星象函数类S_s^*,$f \in S_s^*$当且仅当

$$\mathrm{Re}\frac{zf'(z)}{f(z) - f(-z)} > 0.$$

1987 年,El-Ashwa 和 Thomas[10]引入并研究了关于共轭点的星象函数类及关于对称共轭点的星象函数类,分别满足如下条件

$$\mathrm{Re}\frac{zf'(z)}{f(z) + \overline{f(\bar{z})}} > 0 \text{ 和 } \mathrm{Re}\frac{zf'(z)}{f(z) - \overline{f(-\bar{z})}} > 0.$$

本文将研究一类具有对称共轭点的亚纯双向单叶倒星象函数类如下,

定义 1.1　函数$f(z) \in \Sigma$属于关于对称共轭点的亚纯双向单叶倒星象函数类$\Sigma S_{sc}(\varphi)$当且仅当

$$\frac{-1}{1-\beta}\left\{\frac{f(z) - \overline{f(-\bar{z})}}{2zf'(z)} + \beta\right\} \prec \varphi(z), \tag{3}$$

及

$$\frac{-1}{1-\beta}\left\{\frac{g(\omega) - \overline{g(-\bar{\omega})}}{2\omega g'(\omega)} + \beta\right\} \prec \varphi(\omega), \tag{4}$$

其中$g(\omega) = f^{-1}(\omega)$,$\varphi(z) = 1 + B_1 z + B_2 z^2 + \cdots$,$B_1 > 0$,$\beta < 1$.

2　积分表达式

首先,定义的函数类的积分表达式. 所得结论推广了亚纯p叶函数类的一般已得到的积分表达式[11-13].

定理 2.1　若$f(z) \in \Sigma S_{sc}(\varphi)$,则

$$f(z)=\frac{\exp\int_0^z\frac{(1-\beta)}{2t}\left[\frac{\varphi(u(t))-1}{(1-\beta)\varphi(u(t))+\beta}+\frac{\overline{\varphi(u(-\overline{t}))}-1}{(1-\beta)\overline{\varphi(u(-\overline{t}))}+\beta}\right]dt}{z[(1-\beta)\varphi(u(z))+\beta]}*[z^{-1}+\log(1-z)],\tag{5}$$

其中 $u(z)$ 在 U 内解析且 $u(0)=0$ 及 $|u(z)|<1$.

证明:因为 $f(z)\in\Sigma S_{sc}(\varphi)$,则

$$\frac{-1}{1-\beta}\left\{\frac{f(z)-\overline{f(-\overline{z})}}{2zf'(z)}+\beta\right\}\prec\varphi(z),$$

及

$$\frac{-1}{1-\beta}\left\{\frac{g(\omega)-\overline{g(-\overline{\omega})}}{2\omega g'(\omega)}+\beta\right\}\prec\varphi(\omega).$$

根据从属关系定义,存在解析函数 $u,v:U\to U$ 满足 $u(0)=v(0)=0$, $|u(z)|<1$ 及 $|v(\omega)|<1$,使得

$$\frac{-1}{1-\beta}\left\{\frac{f(z)-\overline{f(-\overline{z})}}{2zf'(z)}+\beta\right\}=\varphi(u(z)),\tag{6}$$

及

$$\frac{-1}{1-\beta}\left\{\frac{g(\omega)-\overline{g(-\overline{\omega})}}{2\omega g'(\omega)}+\beta\right\}=\varphi(v(\omega)).\tag{7}$$

(6)式等价于:

$$\frac{2zf'(z)}{f(z)-\overline{f(-\overline{z})}}=\frac{-1}{(1-\beta)\varphi(u(z))+\beta}.\tag{8}$$

用 $-\overline{z}$ 替代(8)式中的 z 且等式两边取共轭,得

$$\frac{2z\overline{f'(-\overline{z})}}{f(z)-\overline{f(-\overline{z})}}=\frac{-1}{(1-\beta)\overline{\varphi(u(-\overline{z}))}+\beta}.\tag{9}$$

根据(8)和(9)及 $f'(z)+\overline{f'(-\overline{z})}=(f(z)-\overline{f(-\overline{z})})'$,有

$$\frac{z(f(z)-\overline{f(-\overline{z})})'}{f(z)-\overline{f(-\overline{z})}}=\frac{1}{2}\left\{\frac{-1}{(1-\beta)\varphi(u(z))+\beta}+\frac{-1}{(1-\beta)\overline{\varphi(u(-\overline{z}))}+\beta}\right\}.$$

整理后,得

$$\frac{z(f(z)-\overline{f(-\overline{z})})'}{f(z)-\overline{f(-\overline{z})}}+\frac{1}{z}=\frac{1-\beta}{2z}\left\{\frac{\varphi(u(z))-1}{(1-\beta)\varphi(u(z))+\beta}+\frac{\overline{\varphi(u(-\overline{z}))}-1}{(1-\beta)\overline{\varphi(u(-\overline{z}))}+\beta}\right\}.\tag{10}$$

对等式(10)两边积分,得

$$\log\left\{\frac{f(z)-\overline{f(-\overline{z})}}{2}\cdot z\right\}=\int_0^z\varphi_\beta^u(t)\,dt,$$

其中

$$\varphi_\beta^u(t)=\frac{1-\beta}{2t}\left\{\frac{\varphi(u(t))-1}{(1-\beta)\varphi(u(t))+\beta}+\frac{\overline{\varphi(u(-\overline{t}))}-1}{(1-\beta)\overline{\varphi(u(-\overline{t}))}+\beta}\right\}.\tag{11}$$

从而

$$\frac{f(z)-\overline{f(-\bar{z})}}{2}=z^{-1}exp\int_0^z\varphi_\beta^u(t)dt. \tag{12}$$

根据(8)和(12)式,有

$$-zf'(z)=\frac{-1}{z[(1-\beta)\varphi(u(z))+\beta]}exp\int_0^z\varphi_\beta^u(t)dt. \tag{13}$$

根据 Hadamard 积(卷积)的性质,有

$$f(z)=-zf'(z)*[z^{-1}+\ln(1-z)]. \tag{14}$$

因此,利用式(11),(13)及(14),有

$$f(z)=\frac{exp\int_0^z\frac{(1-\beta)}{2t}\left[\frac{\varphi(u(t))-1}{(1-\beta)\varphi(u(t))+\beta}+\frac{\overline{\varphi(u(-\bar{t}))}-1}{(1-\beta)\overline{\varphi(u(-\bar{t}))}+\beta}\right]dt}{z[(1-\beta)\varphi(u(z))+\beta]}*[z^{-1}+\log(1-z)].$$

从而定理 2.1 得证.

3　系数估计

引理 3.1[14]　如果 $p(z)=1+\sum_{n=1}^{\infty}c_nz^n\in P$,则

$$|c_n|\le 2,n=1,2,\cdots.$$

上面的不等式估计是精确的. 当函数 $p(z)=\frac{1+z}{1-z}$ 时,不等式的等号成立.

引理 3.2[15-16]　如果 $p(z)=1+\sum_{n=1}^{\infty}c_nz^n\in P$,则存在复数 x,y,且 $|x|\le 1,|y|\le 1$,使得

$$2c_2=c_1^2+x(4-c_1^2),$$

$$4c_3=c_1^3+2c_1(4-c_1^2)x-c_1(4-c_1^2)x^2+2(4-c_1^2)(1-|x|^2)y.$$

定理 3.1　函数 $f(z)$ 具有(1)式形式,若 $f(z)\in\Sigma S_{sc}(\varphi)$,则有如下系数估计

$$|a_1|\le\frac{(1-\beta)B_1}{2},\ |a_2|\le\frac{(1-\beta)B_1}{2},$$

及

$$|a_2-\mu a_1^2|\le\begin{cases}\frac{(1-\beta)B_1}{2}, & |\mu|\le\frac{2}{(1-\beta)B_1},\\ \frac{|\mu|(1-\beta)^2B_1^2}{4}, & |\mu|>\frac{2}{(1-\beta)B_1}.\end{cases}$$

上面估计是精确的.

证明:因为 $f(z)\in\Sigma S_{sc}(\varphi)$,根据定义 1.1 及根据从属关系定义,存在解析函数 $u,v:U\to U$ 满足 $u(0)=v(0)=0$,$|u(z)|<1$ 及 $|v(\omega)|<1$,使得

$$\frac{-1}{1-\beta}\left\{\frac{f(z)-\overline{f(-\bar{z})}}{2zf'(z)}+\beta\right\}=\varphi(u(z)), \tag{15}$$

及

$$\frac{-1}{1-\beta}\left\{\frac{g(\omega)-\overline{g(-\bar{\omega})}}{2\omega g'(\omega)}+\beta\right\}=\varphi(v(\omega)), \tag{16}$$

令

$$F(z):=\frac{-1}{1-\beta}\left\{\frac{f(z)-\overline{f(-\bar{z})}}{2zf'(z)}+\beta\right\},$$

及

$$G(\omega):=\frac{-1}{1-\beta}\left\{\frac{g(\omega)-\overline{g(-\bar{\omega})}}{2\omega g'(\omega)}+\beta\right\},$$

从而有

$$F(z)=\frac{1+\frac{1+\beta}{1-\beta}a_1z^2+\frac{2\beta}{1-\beta}a_2z^3+\frac{1+3\beta}{1-\beta}a_3z^4+\cdots}{1-a_1z^2-2a_2z^3-3a_3z^4+\cdots},\tag{17}$$

$$G(\omega)=\frac{1-\frac{1+\beta}{1-\beta}a_1\omega^2+\frac{2\beta}{1-\beta}a_2\omega^3+\frac{1+3\beta}{1-\beta}a_3\omega^4+\cdots}{1+a_1\omega^2+2a_2\omega^3+3a_3\omega^4+\cdots}.\tag{18}$$

分别定义函数 $p(z)$ 和 $q(\omega)$ 如下

$$p(z)=\frac{1+u(z)}{1-u(z)}=1+p_1z+p_2z^2+\cdots,$$

$$q(\omega)=\frac{1+v(\omega)}{1-v(\omega)}=1+q_1\omega+q_2\omega^2+\cdots,$$

即,

$$u(z)=\frac{p(z)-1}{p(z)+1}=\frac{1}{2}\{p_1z+(p_2-\frac{p_1^2}{2})z^2+[p_3-p_1p_2+\frac{1}{4}p_1^3]z^3+\cdots\},$$

$$v(\omega)=\frac{q(\omega)-1}{q(\omega)+1}=\frac{1}{2}\{q_1\omega+(q_2-\frac{q_1^2}{2})\omega^2+[q_3-q_1q_2+\frac{1}{4}q_1^3]\omega^3+\cdots\}.$$

显然 $p,q\in P$. 从而,有

$$\varphi(u(z))=1+\frac{B_1p_1}{2}z+[\frac{B_1}{2}(p_2-\frac{1}{2}p_1^2)+\frac{B_2}{4}p_1^2]z^2+\{\frac{B_1}{2}[p_3-p_1p_2+\frac{1}{4}p_1^3]+\frac{B_2}{2}(p_1p_2-\frac{1}{2}p_1^3)+\frac{B_3}{8}p_1^3\}z^3+\cdots\tag{19}$$

$$\varphi(v(\omega))=1+\frac{B_1q_1}{2}\omega+[\frac{B_1}{2}(q_2-\frac{1}{2}q_1^2)+\frac{B_2}{4}q_1^2]\omega^2+\{\frac{B_1}{2}[q_3-q_1q_2+\frac{1}{4}q_1^3]+\frac{B_2}{2}(q_1q_2-\frac{1}{2}q_1^3)+\frac{B_3}{8}q_1^3\}\omega^3+\cdots\tag{20}$$

根据式(15)-(20),得 $p_1=q_1=0$,及

$$a_1=\frac{(1-\beta)B_1}{4}p_2,\tag{21}$$

$$a_2=\frac{(1-\beta)B_1}{4}p_3,\tag{22}$$

$$a_1=-\frac{(1-\beta)B_1}{4}q_2,\tag{23}$$

$$a_2=-\frac{(1-\beta)B_1}{4}q_3.\tag{24}$$

因此,有 $p_2=-q_2$, $p_3=-q_3$.

利用引理 3.1,得

$$|a_1|\le\frac{(1-\beta)B_1}{2}\quad 及\quad |a_2|\le\frac{(1-\beta)B_1}{2}.$$

另一方面,

$$a_2-\mu a_1^2=\frac{(1-\beta)B_1}{4}(p_3-\frac{\mu(1-\beta)B_1}{4}p_2^2).\tag{25}$$

根据引理 3.2,存在复数 x,y,满足 $|x|\le 1$,$|y|\le 1$,使得 $p_2=2x$,$p_3=2(1-|x|^2)y$,则(25)式得

$$a_2-\mu a_1^2=\frac{(1-\beta)B_1}{4}[2(1-|x|^2)y-\mu(1-\beta)B_1x^2].$$

令 $x=re^{i\theta}$,$y=\rho e^{i\varphi}$,$0\le r\le 1$,$0\le\rho\le 1$,$\theta\in[0,2\pi)$,$\varphi\in[0,2\pi)$,则有

$$a_2-\mu a_1^2=\frac{(1-\beta)B_1}{4}[2(1-r^2)\rho e^{i\varphi}-\mu(1-\beta)B_1r^2e^{2i\theta}].$$

于是,

$$|a_2-\mu a_1^2|=\frac{(1-\beta)B_1}{4}\sqrt{4(1-r^2)^2\rho^2-4(1-\beta)\mu B_1(1-r^2)r^2\rho\cos(2\theta-\varphi)+(1-\beta)^2\mu^2B_1^2r^4}\le$$

$$\frac{(1-\beta)B_1}{4}[2(1-r^2)\rho+(1-\beta)B_1|\mu|r^2]=\frac{(1-\beta)B_1}{4}F(\rho,r),$$

其中 $F(\rho,r)=2(1-r^2)\rho+(1-\beta)B_1|\mu|r^2$.

由于 $\frac{\partial F(\rho,r)}{\partial\rho}=2(1-r^2)\ge 0$,从而函数 $F(\rho,r)$ 是关于 ρ 的增函数,于是

$$F(\rho,r)\le F(1,r)=2+[(1-\beta)B_1|\mu|-2]r^2.$$

当 $|\mu|\le\frac{2}{(1-\beta)B_1}$ 时,

$$|a_2-\mu a_1^2|\le\frac{(1-\beta)B_1}{4}\{2+[(1-\beta)B_1|\mu|-2]r^2\}\le\frac{(1-\beta)B_1}{2},$$

当 $|\mu|>\frac{2}{(1-\beta)B_1}$ 时,

$$|a_2-\mu a_1^2|\le\frac{(1-\beta)B_1}{4}\{2+[(1-\beta)B_1|\mu|-2]r^2\}\le$$

$$\frac{(1-\beta)B_1}{4}\{2+[(1-\beta)B_1|\mu|-2]\}=\frac{|\mu|(1-\beta)^2B_1^2}{4}.$$

从而,有

$$|a_2-\mu a_1^2|\le\begin{cases}\frac{(1-\beta)B_1}{2}, & |\mu|\le\frac{2}{(1-\beta)B_1},\\ \frac{|\mu|(1-\beta)^2B_1^2}{4}, & |\mu|>\frac{2}{(1-\beta)B_1}.\end{cases}$$

特别的,当 $f(z)=f_1(z)$ 时,$|a_2-\mu a_1^2|$ 取得极值 $\frac{(1-\beta)B_1}{2}$;当 $f(z)=f_2(z)$ 时,$|a_2-\mu a_1^2|$ 取得极值 $\frac{|\mu|(1-\beta)^2B_1^2}{4}$,其中

$$f_1(z)=\frac{\exp\int_0^z\frac{(1-\beta)}{2t}\left[\frac{\varphi(t^3)-1}{(1-\beta)\varphi(t^3)+\beta}+\frac{\overline{\varphi}(-(\overline{t})^3)-1}{(1-\beta)\overline{\varphi}(-(\overline{t})^3)+\beta}\right]dt}{z[(1-\beta)\varphi(z^3)+\beta]}*[z^{-1}+\log(1-z)].$$

及

$$f_2(z)=\frac{\exp\int_0^z\frac{(1-\beta)}{2t}\left[\frac{\varphi(t^2)-1}{(1-\beta)\varphi(t^2)+\beta}+\frac{\overline{\varphi}((\overline{t})^2)-1}{(1-\beta)\overline{\varphi}((\overline{t})^2)+\beta}\right]dt}{z[(1-\beta)\varphi(z^2)+\beta]}*[z^{-1}+\log(1-z)].$$

推论 3.1 函数$f(z)$具有(1)式形式,若$f(z)\in \Sigma S_{sc}(\frac{1+Az}{1+Bz})$,其中$-1\le B<A\le 1$,则有如下系数估计

$$|a_1|\le\frac{(1-\beta)(A-B)}{2},\quad |a_2|\le\frac{(1-\beta)(A-B)}{2},\text{及}$$

$$|a_2-\mu a_1^2|\le\begin{cases}\frac{(1-\beta)(A-B)}{2}, & |\mu|\le\frac{2}{(1-\beta)(A-B)},\\ \frac{|\mu|(1-\beta)^2(A-B)^2}{4}, & |\mu|>\frac{2}{(1-\beta)(A-B)}.\end{cases}$$

上面估计是精确的.

特别的,当$f(z)=f_1(z)$时,$|a_2-\mu a_1^2|$取得极值$\frac{(1-\beta)(A-B)}{2}$;当$f(z)=f_2(z)$时,$|a_2-\mu a_1^2|$取得极值$\frac{|\mu|(1-\beta)^2(A-B)^2}{4}$,其中

$$f_1(z)=\frac{(1+Bz^3)\{1-[(1-\beta)A+\beta B]^2z^6\}^{\frac{(1-\beta)(A-B)}{6[(1-\beta)A+\beta B]}}}{z\{1+[(1-\beta)A+\beta B]z^3\}}*[z^{-1}+\log(1-z)],$$

$$f_2(z)=\{\frac{1}{z}(1+Bz^2)\{1+[(1-\beta)A+\beta B]z^2\}^{\frac{(1-\beta)(A-B)}{2[(1-\beta)A+\beta B]}-1}\}*[z^{-1}+\log(1-z)].$$

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